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ON THE UNION OF MATTER AND FORM: GROUP THEORETICAL APPROACH

V.V. Varlamov

Siberian State Industrial University
42 building, Kirov St., Central district, Novokuznetsk,
Kemerovo region – Kuzbass, 654007 Russian Federation

Abstract. What are space and time? What do elementary particles and elements of the periodic table have in common? These and other equally intriguing questions are discussed in this interview with Professor of Siberian State industrial University Vadim Varlamov.

Keywords: holism, reductionism, hylomorphism, spinors, elementary length, symmetries, long-range action

Q: Dear Professor Varlamov, thank you very much for taking your time to do this interview. Let us begin from the fundamentals. Physicists today are often in doubt whether spacetime is the deepest level of reality. The level under spacetime is quantum. It is essentially discrete, while the continuity of spacetime emerges for us in the same manner as that from an offset print, a pixelized monitor image, or impressionist painting. What is your attitude towards impressionism in this sense?

A.: Does spacetime make the fundamental level of reality? First define what are these “space” and “time” comprising the notion of spacetime. Two different frameworks should be distinguished: aprioristic Kantian understanding of space and time as pure forms of our sensibility and the so-called relativistic view on space and time as the forms of existence of matter. Forms of sensibility versus forms of being. Consciousness versus matter. It seems like we are trapped once again within the eternal duality of spirit and matter - nightmare of a philosopher having to choose between the idealistic and materialistic philosophies. As a matter of fact, this duality is only an illusion caused by the mess of words and concepts, as was noted by Bertran Russel. Let us take a closer look at the two key concepts of space and time. Most generally, 3d space is an extension with three parameters (length, width, height); and time is a one-way ordered duration (from past via present to future). Anyhow, these are our common representations of space and time and their essential definitions, which are inevitably tied with the human mind. These two concepts cannot be deduced empirically but precede any experiment making the ground for all the following constructions. According to Kant, space and time are two pure forms of sensory contemplation and principles of a priori knowledge. They are the first and fundamental attributes of our perception, derived from the integrated sensorial experience of human being. Hume treated each of us as “nothing but a bundle or collection of different perceptions, which succeed each other with an inconceivable rapidity”.

Form of contemplation is just an obscure and strongly curved lens mediating our perception of the world. Hence, defining the space and time concepts from experience makes no sense, while defining them by other means yields anything but not space and not time. Here we come to a very important conclusion: *space and time are nothing but forms of human conscious contemplation*.

The next fundamental concept that we turn now is tightly connected with the above and highly important for the following. The key mathematical structure associated with space and time is continuum, but *whether it is sufficient to describe the Universe?* Is it possible to describe all the phenomena of Nature within the Minkowski framework or only a part of them, and what is this part then? In the opinion of Reihenbach, “… the combined space-time order reveals itself as the ordering schema governing causal chains and thus as the expression of the causal structure of the universe” [1, p. 268].

So, continuum expresses the causal structure of the universe. However, the causal structure is not enough to describe the whole universe because of the worlds that are causally independent (the fact acknowledged by the scientific society today) and exist simultaneously («simultaneity means the exclusion of causal connection» [1, p. 145)]). A puzzling but necessary conclusion is that such worlds exist beyond of time.

After the discovery of non-Euclidean geometries, multidimensional spaces, and hypercomplex algebras late in XIX, mathematics experienced a cardinal transformation. The three-dimensional (3d) Euclidean geometry reflecting the sensorily perceived world (form of contemplation) appeared to be only a preliminary stage in the long mathematical venture.

The absolutized and iconized 3d Euclidean space appeared to be only the simplest case of much more complicated mathematical structures. The mathematical notion of space had changed beyond recognition. Is there anything in common between the intricate constructions called “spaces” by mathematicians and the routine notion of space we have in mind? This word is only a tribute to tradition, it had stepped beyond the original meaning as the form of perception and now symbolizes notions of quite a different nature. For example, in the theory of particles, quarks live in the spaces of aroma, strangeness, charm, and so on. What are these abstract mathematical spaces and do they still have something in common with forms of our contemplation? We will pursue the matter below.

The most general mathematical abstraction of space is topological space. Let us recall the definition. Given a set X of arbitrary nature, specify a collection of subsets $\tau=\{U\}$ with the following properties:

1) τ includes the empty set and set X itself;

2) the union of any collection of sets from τ belongs to τ;

3) the intersection of any finite number of sets from τ belongs to τ.

Such a collection of subsets τ is known as *topology* on X, and the set X itself is *topological space* [2, p. 41]. The notions of point neighborhood, base of topology, axioms of countability and separability constitute the mathematical substrate of space. The notions of dimension, connectivity, metrics, signature, curvature, etc., arise later; these structures are of secondary, derivative nature. For example, dimension in the Urysohn-Menger topological theory derivates from the space topology.

The above given definition of topological space sounds a bit vague at first glance but a deeper insight shows that it has a core notion which has a clear correlate in our perception. Recall what the Hausdorff separation axiom says: any two different points x and y of topological space have nonintersecting neighborhoods. *Extension* is the key word which expresses the very essence of the space substrate! Indeed, the notions of dimension, connectivity, metrics (distance), curvature do not make sense for anything nonextended. All of them are attributes of an extended value and, moreover, extension is the primary and necessary condition of their existence. Mind a curious thought of Borges: "The line is made up of an infinite number of points; the plane of an infinite number of lines; the volume of an infinite number of planes; the hypervolume of an infinite number of volumes. No, unquestionably this is not - more geometrico - the best way of beginning my story. To claim that is it true is nowadays the convention of every made-up story. Mine, however, is true" [3, p. 117].

In this witty phrase Borges does not take account of the fact that each of the points consisting a line should be included there together with its neighborhood. It is precisely this requirement (to take the point neighborhoods into account) that lies in the core of the Urysohn-Menger topological theory. Similarly, our common experience tells us that the extension of the world around is the necessary condition and first form-making factor of the phenomenological world that we perceive. The first and the most important derivative of extension is *dimension.*

Time

Time is one of the greatest enigmas of science and culture, the importance of which extends far beyond the scope of physics. There is hardly another notion carrying such a heavy psychological load and having such a wide spectrum of interpretations. In his brilliant essay «A History of Eternity» Borges writes: «For us, time is a jarring, urgent problem, perhaps the most vital problem of metaphysics, while eternity is a game or a spent hope». [4, p. 1].

At first glance the concept of time seems simpler than that of space: «Time therefore seems to be much less problematic since it has none of the difficulties resulting from multi-dimensionality» [1, p. 109].

We conceive time as a linear one-dimensional sequence either lasting forever towards past and future or cycling like arrows on the clock face or rotating planets. This is our immanent sense of time as duration. Trying to grasp the essence of time Heidegger wrote: «We say "now" and mean time. But time cannot be found anywhere in the watch that indicates time, neither on the dial nor in the mechanism, nor can it be found in modern technological chronometers. The assertion forces itself upon us: the more technological, the more exact and informative the chronometer, the less occasion to give thought first of all to time's peculiar character. But where is time? Is time at all and does it have a place?» [5, p. 11].

Such is the form of contemplation in which time appears to our mind. A naive realist being unaware of the form of perception identifies it with the real being thus endowing it with ontological status. In this way, the *substantial time* arises, considered as some absolute existing in the universe: «*Absolute, true, and mathematical time*, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration» [6, p. 77].

An alternative concept goes back to Leibnitz and is known as the *relational time*. In brief, it states that time is the sequence order of events. This definition is naturally inherent to both the special and general relativity theories. Relativity theory admits neither a preferred reference system nor absolute, universal time. Relative time expresses causal order of events occurring in the universe and has no meaning regardless of this order. «Newtonian time was universal; relativistic time is, at it were, personal. My time is not your time nor your time my time» says Synge [7, p. 102].

Moreover, the time of special relativity depends on spatial coordinates, as seen from the Lorentz transform. Time and space merge within the four-dimensional continuum of Minkowski spacetime thus losing their independence and autonomy. "Time is a sequence order", - says Leibnitz. Indeed, taking off the emotional and psychological stratifications on the concept of time, we are left with the key word of '*sequence*'. In "Time and J. W. Dunne" Borges said that he does not "pretend to know what sort of thing time is — or even if it is a 'thing'; however, he guessed that the time itself and the order of time is a *one* mystery and not *two*" [8]. At this point, we should make the following remark concerning the sequence order, or the order of time, although *«anyway time-order is not time»* [7, p. 103].

Considering the known idea of *time arrow*, one should keep in mind that time passes one-way at the macroscopic scale only. Due to Reihenbach, this directivity of time is related to thermodynamic *entropy*, which reflects the irreversible character of macroscopic processes [9]. However, if we change scale to subatomic values, the sequence order does not need to be one-way any more. In accordance with famous Feynman-Stueckelberg interpretation, particles moving back in time are antiparticles, i.e., their sequence order is reverse. Borges also admits time reversal: "I will begin by listing some of the obscurities inherent in time, a natural, metaphysical mystery that must precede eternity, which is a daughter of mankind. One such obscurity, neither the most challenging nor the least beautiful, keeps us from ascertaining the direction in which time moves. It is commonly held to flow from past to future, but the opposite notion, established in Spanish verse by Miguel de Unamuno, is no less logical: Nocturno el rio de las horas fluye desde su manatial que es el manana eterno ..." [Nocturnal the river of hours flows from its source, the eternal tomorrow ...] [4, p. 1]. Such a notion of time is close to the views of scholastics who conceived it as motion from potential to reality.

Sequence is the dynamical aspect of continuum. *A sequence given in extension generates causality*. Causality is the synthesis of sequence and extension, their one and nonseparable union. Indeed, extension without sequence is static, there is no dynamics and, hence, no causality. In turn, sequence without extension is only a potential of causality, which comes into being provided the stage, which is extension. Therefore, the formula "space-and-time = continuum" with its left side containing forms of a priori contemplation (archaic words doomed to be a fiction by Minkowski) should be replaced by "extension-and-duration = causality". Whence it follows, continuum = causality.

Time exists only together with extension, i.e., under the separability and countability conditions providing a separable space. In nonseparable spaces, where topology has no countable base, the concepts of sequence and time are out of place.

Q.: An assembled mechanism is evidently not the same as the set of its parts. In spite of this evidence, impelled by the reductional idea we continue to disassemble matter into ever small parts, has already come to quarks and intend to proceed. In recent papers you assert that the "spinor structure is more fundamental that the quark one". Please explain what do you mean - is there anything still more fundamental than quarks and what are these weird spinors then?

A.: Mathematically, the quark and spinor structures are determined by the tensor product machinery, which implies different number of basic components in these two cases: $n = 2$ for spinors and $n = 3$ for quarks; this difference suggests that *spinors are more fundamental than quarks*.

Concerning the essential definition of spinors, Michael Atiyah once said: "No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious".

Spinors can be defined either geometrically (due to Cartan) or algebraically (due to Brauer and Weyl). According to Cartan, spinors make up vector spaces where the linear transformations of vectors represent motions in non-Euclidean spaces S^n (motion is an isometry of a metric space). The vectors of representation space are called spinors of space S^n. This concept was introduced by Cartan [10], while the word 'spinor' was coined by Ehrenfest after he read the famous paper on 'spinning electron' by Uhlenbeck and Goudsmit [12]. More precisely, the spinor representations of motions in non-Euclidean spaces S^n implies viewing the spinor coordinates as coordinates of the flat generatrices of the absolutes (absolute is the set of infinitely far points of non-Euclidean space) of these spaces; the spinor transformations corresponding to the transformations of absolutes under the motions are the spinor representations of these motions.

In case important for physics, the group of motions in 3d non-Euclidean space $S^{1,2}$ (Lobachevsky space) is isomorphic to the group of rotations in 4d pseudo-Euclidean Minkowski spacetime $R^{1,3}$, which is also the Lorentz group of the special relativity. Therefore, the spinor representation of the group of motions in $S^{1,2}$ (by 2x2 complex matrices with det = 1) is also the spinor representation of the Lorentz group. This means that each spinor from $S^{1,2}$ corresponds to a certain point of the absolute of $S^{1,2}$ which in turn corresponds to an isotropic straight line of space $R^{1,3}$. The absolute of the Lobachevsky space $S^{1,2}$ is homeomorphic to the compactified complex plane $\mathbb{C} \cup \infty$ (Riemannian sphere). This geometrical interpretation of spinors was proposed by Cartan [13] (see also [14]).

In the algebraic Brauer-Weyl definition, spinor is given by an element of the minimal left ideal of the Clifford algebra $cl(V, Q)$, where V is the vector space equipped with a nondegenerate quadratic form Q. For even n, the minimal left ideal of algebra $cl(V, Q)$ corresponds to the *maximal totally isotropic subspace* $U \subset V$ of dimension $n/2$, i.e., isomorphic to spin space $\mathbb{S}$ of dimension $2^{n/2}$. Subspace U of space V is totally isotropic if bilinear form $B(\alpha_i, \alpha_j) = 0$ for every $\alpha_i, \alpha_j \in U$. Subspace $U \subset V$ of the maximal dimension satisfying this condition is the maximal totally isotropic space.

The mathematical notion of spinor, which we defined geometrically and algebraically, is closely connected with the physical concept of spin. The main puzzle is in the physics of spin rather than in the geometry of spinors as Atiyah thought. It is known that the notion of spin was coined by Pauli in 1925, when he tried to explain the doublet structure of the alkali spectra (anomalous Zeeman effect): «The doublet structure of the alkali spectra, as well as the violation of the Larmor theorem are, according to this point of view, a result of a classically not describable two-valuedness of the quantum-theoretical properties of the valence electron” [14]. Later, Van der Waerden noted that this “classically non-describable two-valuedness” of electron we call spin [11, p. 215].

It is well-known that, starting from the work of Uhlenbeck and Goudsmit [12], all the attempts to describe the electron spin classically had failed. In memorials Yu. B. Rumer [16] wrote: “At his time Pauli said to Kronig that the spin theory is nonsense because a point cannot rotate around itself” [16, p. 56]. Viewing electron as a point is required in the special relativity; therefore, both electrons in quantum electrodynamics and quarks in quantum chromodynamics are treated as pointwise fermions with spin 1/2. Furthermore, the quark model ascribes quarks the orbital and radial motions inside hadrons. The hadron spin is associated with the total momentum $\boldsymbol{J}$, which is the sum of the orbital momentum $\boldsymbol{L}$ and the quark spin $\boldsymbol{S}$: $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. This definition is evidently a mechanical parody of the real spin. According to Fock [17], *the concept of spin is not of mechanical character*. The first theory providing a correct mathematical formulation of the “classically non-describable two-valuedness” of the electron spin was proposed by Pauli in 1927 [17]. Avoiding any visual mechanical models, Pauli introduced doubled Hilbert space $\boldsymbol{H}_2 \otimes \boldsymbol{H}_\infty$ (vector space of wave functions), whose vectors are two-component spinors. That was the first advent of two-component spinors in physics and the first case of doubling. The next doubling that yielded the $\boldsymbol{H}_4 \otimes \boldsymbol{H}_\infty$ space of *bispinors* was accomplished by Dirac in 1928 [19]. The next-to-next doubling leads to the Hilbert space of *hyper-twistors* $\boldsymbol{H}_8 \otimes \boldsymbol{H}_\infty$ [20,21]. Hyper-twistors are vectors of the fundamental representation of the Rumer-Fet group $SO(2,4) \otimes SU(2) \otimes SU(2)'$, which provides a group-theoretic interpretation of the periodic system of elements [22]. Spinors, bispinors, and twistors are special cases of hyper-twistors. Van der Waarden mostly appreciated the first doubling step: «The step from one to two to components is large, whereas the step from two to four components is small; also, the step from vector algebra to a two-valued representation of the rotation group is large, the extension of this representation to the Lorentz group is much easier» [11, p. 223].

After reporting on the doublet splitting value by Thomas [23], Pauli formally accepted the hypothesis of rotating electron but stayed convinced that spin cannot be described by a classical kinematic model since such a model could never lead to two-valued representations of the rotation group. This compromise of theorists about spin Pauli expressed by the words: «After a brief period of spiritual and human confusion, caused by a provisional restriction to "Anschaulichkeit", a general agreement was reached following the substitution of abstract mathematical symbols, as for instance psi, for concrete pictures. Especially the concrete picture of rotation has been replaced by mathematical characteristics of the representations of the group of rotations in three-dimensional space. This group was soon amplified to the Lorentz group in the work of Dirac» [11, p. 5]. The most important is the last sentence because the correct description of spin requires two-valued representations of the Lorentz group and cannot be provided by the traditional description of spin in terms of $SU(2)$ group appealing to visual 3d image. The $SU(2)$-handling of spin is adopted in the Standard Model, in the $SU(6)$-theory, and its orbital extension $SU(6) \otimes O(3)$, as well as in the notion of isotopic spin (“rotation” in the isotopic space). The $SU(2)$-description reduced to the 3d image (the compromise explained by Pauli) does not capture all the properties of spin, which is the most important characteristic of a quantum object (a state). For example, the $SU(2)$-framework gives no way for two states to have equal spins but different masses (energies). However, this turns possible in terms of the $SL(2,\mathbb{C})$-model, where spin $s = l - \dot{l}$, $l = k/2$, $\dot{l} = r/2$, k and r are degrees of spin-tensor factorizations of cyclic vectors defining fermionic and bosonic states. The $SL(2,\mathbb{C})$-description of spin can be found in [25, 26].

Recall that the concept of spin generally stemmed from the need to describe the “classically non-describable two-valuedness” of the electron *conceived as a particle*. Particle is a classical notion, however. Van der Waerden suspected a confusion here and asked: «Why did Pauli express himself so mysteriously? Why did he not say in plain words that he assumed the electron to have an intrinsic angular momentum

$m_s = \pm 1/2$ and a magnetic moment $2m_s$?" [11, p. 205]. The answer is evident: because a classical object cannot have a nonclassical property. Soviet physicists Yu. B. Rumer and A. I. Fet wrote: "We conventionally consider that *one and the same particle*, e.g., electron may stay in two different states, with spin +1/2 and -1/2. But nobody has ever observed an electron without a spin. This fact suggests another viewpoint: electron with spin +1/2 and electron with spin -1/2 are two different particles, while 'just electron' is not encountered in nature" [27, p. 161-162].

It is commonly acknowledged today that the spacetime incorporates some objects (protons, electrons, etc.) which are independent from each other and possess their own being and all the quantum features inherent to them. Needless to remind that such a view has nothing to do with reality shown up in the experiments on checking the Bell inequalities (Fridman—Clauser, Aspect, Greenberger–Horne–Zeilinger, and others). Classical treatment of microscopic particles is invalid, and so are all the mechanical models related to them. As soon as we get rid of the visual picture (mental pattern) of a ball rotating in space, we realize that the real base of the notion of particle is its discrete nature, the adoption of a smallest further indivisible value or the negation of *continuum*. We arrive at the eternal syzygies of quant and continuum, Democritus and Empedocles. In contrast to purely mental procedure of infinite division that turns continuum into a speculation, the notion of quantum relies on the Planck fundamental law on the discreteness of energy. In the paper "On the contemporary form of atomism" [28], M. A. Markov shows how the conception of particle has been evolving since the ancient time to the modern state and concludes that "today, given the notion of field, we define particle as an atom, or a quantum of this field, its minimal element. In essence, we only changed the word 'atom' for the word 'quantum'". An obscure notion of particle is defined by means of another no less obscure notion of field. Markov himself notices this fact: "Unfortunately, the notion of field is also not easy to explain" [28, p. 412]. Then Markov draws a curious parallel between four types of interactions in contemporary physics and four main elements of ancient Greeks, clearly letting the reader to understand that the former had not gone very far from the latter. Field is an inalienable attribute of the close action because field cannot be defined without spacetime continuum (which facilitates the point-to-point transfer of the action). We should remind that the original notion of 'quantum' had nothing to do with that of 'field'. Moreover, the discrete character of energy contradicts the continuity of continuum. In our opinion, particle is rather a *quantum of energy* than a quantum of field.

Let us return to the spinor representations of the Lorentz group. The *fundamental representation* over the complex numbers $\mathbb{F} = \mathbb{C}$ acts in two-dimensional spin-space; its 'vectors' are two-component spinors. Any arbitrary finitedimensional irreducible spinor representation of the Lorentz group can be expanded into a tensor product of two-dimensional fundamental representations. Remarkably, the spin-space of fundamental representation is the minimal left ideal of the algebra of quaternions (biquaternions $\mathbb{C}_2 \simeq \mathbb{C}\otimes\mathbb{H}$ in case of field $\mathbb{F}=\mathbb{C}$ and quaternions $cl_{0,2} \simeq \mathbb{H}$, $cl_{1,1} \simeq \mathbb{R}(2)$, $cl_{2,0} \simeq \mathbb{R}(2)$ in case of field $\mathbb{F}=\mathbb{R}$, with the division ring $\mathbb{K} \simeq \mathbb{H}$ for $cl_{0,2}$ and $\mathbb{K} \simeq \mathbb{R}$ for $cl_{1,1}$ and $cl_{2,0}$). Thus, the elements of the minimal left ideals of the four-dimensional quaternion algebras are two-component spinors.

Q.: 'In the beginning was symmetry' is a famous Heisenberg's phrase. Symmetry is in the core of field theory today. Isn't the role of symmetry overestimated? Do you agree that the conserved properties are in a sense more real than illusory, ever changing, observable phenomena?

A.: Let me give a bit longer citation of Heisenberg: "'In the beginning was symmetry' is certainly a better expression than Democritus' 'In the beginning was the particle.' Elementary particles embody symmetries; they are their simplest representations, and yet they are merely their consequence" [29, p. 240]. Here, Heisenberg clearly gives preference for the symmetry-based holistic description of quantum things over the reductional atomic hypothesis.

Aristotle said that any physical object is a union of matter and form. During the 19th century, this viewpoint had been developed into hylomorphism (from the Greek $\upsilon\lambda\eta$ (hyle) -- tree, matter, and $\mu o\rho\phi\eta$ (morphe) - form, shape). Aristotle believed in such thing as 'primary matter' which consists of four elements, each resulting from the conjunction of prematter and a special form. Hence, prematter is thought of as a pure potentiality. For Heisenberg, his *prima materia* was the fundamental spinor field, which was the backbone of his spinor theory of matter. Potentiality should be complemented with a form-establishing principle which Heisenberg believed to be the fundamental symmetry. Symmetry endows matter with a form. Together, prima materia and fundamental symmetry give birth to all the quantum microscopic phenomena which we call elementary 'particles'.

It is easy to see that this union of matter and form is another version of the old archetype similar to the Aristotle's hylomorphism. Jung was right saying that all human knowledge consists of just cases of realized archetypes, which eternally exist in the collective unconscious. Most likely, Aristotle was not the first who made up his mind to combine form and matter.

Without claiming priority, after Aristotle and Heisenberg, we propose the following (group-theoretical) version of hylomorphism (see axiom **A.I** in [30]):

A.I (Energy and fundamental symmetry) *At the fundamental level, a quantum system* **U** *is characterized by* C^**-algebra* $\mathfrak{A}$ *consisting of the energy operator* H *and the fundamental symmetry group* G_f *generators adjoint to* H *and forming a common system of eigenfunctions with* H.

Here, energy (operator H) plays the role of the *prima materia* that is initially devoid of form and is only a pure potentiality (of dynamics, $\delta\upsilon\nu\alpha\mu\sigma$). The form is brought in by the fundamental symmetry G_f. This marriage of matter and form (hylomorphism) gives rise to a quantum system **U** whose states are different physical objects depending on G_f (elementary particles when $G_f = SO(1,3)$ is the Lorentz group, and chemical elements when $G_f = SO(2,4)$ is the conformal group). Note that using the length scale $l_0 \sim 10^{-13} cm$ as elementary is not typical of physics today and needs to be explained. Back in 1968, in the paper "Does an elementary length exist?" [31], A. D. Sakharov wrote: "Thus, the sum of theoretical and experimental arguments compels us to accept that the Heisenberg's proposal $l_0 = r$ should be revised towards much higher energies". Since that time, the ultimate value has been shifted to ever deeper range $\ll l_0$. To find new physics at such high-energy scale was the main hope and anticipation of the Large Hadronic Collider (LHC) project. However, despite of the tonnes of experimental data, nothing new was revealed: no supersymmetry, no dark matter, nothing at all that could lead beyond the Standard model. The disappointment resulted among physicists is well described by Sabina Hossenfelder in her "Lost in Math" [32]. No other evidence is required to the fact that scale l_0 is elementary since it is particularly this level where all stable matter emerges from the prima materia. On the contrary, any theory operating at the Planck scale $l_P \sim 10^{-33}\ cm$ (e.g., string theory) inevitably has no chance of being verified.

Q.: In your papers you refer to Pauli who said that a key to symmetry is bipartition, or doubling. Could you explain what Pauli really meant: how can doubling give any sort of symmetry, for example, a continuous one? Does doubling result in a growth or rather in a reduction of symmetry?

A.: Indeed, in one of his letters to Heisenberg, Pauli wrote about the doubling-up process and the reduction of symmetry "Das ist des Pudels Kern"! What could it mean? This puzzling phrase relates to one of the most mysterious events in physics of the XX century. The story began in 1957, when Heisenberg and Pauli came to Padua to take part in a conference on atomic physics. Later on, Heisenberg recalled that time: "Chief among these was the work of the young Chinese American physicists Lee and Yang. These two had put forward the suggestion that mirror or right-left symmetry, always considered an almost self-evident part of nature, could be disturbed by such weak interactions as are, for example, responsible for radioactive phenomena" [29, p. 230]. The assumption of Lee and Yang (1956) that nature adopts only left-handed neutrinos and right-handed antineutrinos was later verified in the famous experiments of Wu. Heisenberg continues: "Now, if these particles were indeed devoid of mirror symmetry, then we had to reckon with the possibility that the latter was not a primary aspect of the fundamental laws of nature but entered: them secondarily by way of, for example, interactions and the resulting mass. In that case, mirror symmetry would originate in a subsequent doubling-up process that could arise mathematically, for instance, through the fact that an equation admits of two equivalent solutions. This possibility seemed very exciting to us, simply because it amounted to a simplification of the fundamental laws of nature" [29, p. 230-231].

So, the place to start from was the reduction of symmetry observed for neutrino, which in Heisenberg's opinion can be restored by the doubling-up process, i.e. by finding a pair of solutions to a certain equation. Such an equation laid a basis for the *unified nonlinear spinor theory of matter* developed by Heisenberg late in 1957: «Quite suddenly, there appeared among the fluctuating forms a field equation with a very high degree of symmetry» [29, p. 233].

This equation (the 'world formula') is a nonlinear generalization of the Dirac equation under the assumption that mass has a field nature. A solution (wave function) to that equation should be given by a fundamental spinor field (prima materia), where the field operator is a two-component Weyl spinor under the Lorentz transformation and a two-component spinor in the isospin space, i.e., obeys not only the relativistic spacetime symmetry but also the isospin group symmetry. Thus, the equation described most part of the symmetries known at that time (1957). Without a delay, Heisenberg informed Pauli about this discovery: "Wolfgang, whom I informed of the latest development, was extremely excited as well: it really did look as if, for the first time, we had a framework wide enough to include the entire spectrum of elementary particles and their interactions, and yet narrow enough to determine everything in this field apart from contingent factors. And so we decided that both of us would look into the question of whether or not this equation might serve as a basis for a unified field theory of elementary particles. Wolfgang was hopeful that what few symmetries were still missing might be added later by means of the division process" [29, p. 233].

From here on the most strange and intriguing part of the story begins. At first Heisenberg notes: "With every step Wolfgang took in this direction, he became more enthusiastic—never before or afterward have I seen him so excited about physics" [29, p. 233]. Then, he continues: "For instance, in the theory of elementary particles he waxes enthusiastic over the different four-term symmetries interlaced with each other which he immediately relates to the tetractys of the Pythagoreans. Again, he writes: "Bipartition and reduction in symmetry, that is the core of the matter (des Pudels Kern). Bipartition is a very old attribute of the devil (the word doubt is supposed to have meant originally division into two)"" [33, p. 53].

These weird images were inspired by the Jungian archetypes (Pauli was in correspondence with Jung for many years [34]). It is well known that one source of the Jungian psychoanalysis was the holistic view of the universe [35]. For Pauli, of most interest in correspondence with Jung was the transition from duality 2×6 to quaternity 3×4: "For this magical view of nature the predominant symbol is the number four, the so-called tetractys of the Pythagoreans which is constructed by means of two polarities. Division is attributed to the dark side of the world (matter, the devil), and the magical conception of nature encompasses even this dark realm" [33, p. 52].

Earlier, in 1952, Pauli wrote an unexpected of him paper "The Influence of Archetypical Ideas on the Scientific Theories of Kepler" [36], where he analyzed the polemics between Kepler and Fludd (a known at that time alchemist and a member of the Order of the Rose). Pauli emphasized the importance of the Pythagorean tetractys: "It is significant for the psychological contrast between Kepler and Fludd that for Fludd the number four has a special symbolical character, which, as we have seen, is not true of Kepler" [36, p. 204]. And below: "… that the 'quaternary' attitude of Fludd corresponds, in contrast to Kepler's 'trinitarian' attitude, from a psychological point of view, to a greater completeness of experience" [36, p. 206].

Although Pauli did not generally approve of the Fludd's alchemy and astrology but evidently sympathized him: "Fludd's attitude, however, seems to us somewhat easier to understand when it is viewed in the perspective of a more general differentiation between two types of mind, a differentiation that can be traced throughout history, the one type considering the quantitative relations of the parts to be essential, the other the qualitative indivisibility of the whole. We already find this contrast, for example, in antiquity in the two corresponding definitions of beauty: in the one it is the proper agreement of the parts with each other and with the whole, in the other (going back to Plotinus) there is no reference to parts but beauty is the eternal radiance of the 'One' shining through the material phenomenon" [36, p. 205].

In one of his letters to Jung, Pauli wrote : "I carry 'Kepler' as well as 'Fludd' in myself and that it is for me a necessity to arrive at a synthesis of this pair of opposites, as best I can" [37, p. 421].

When writing about Kepler, Pauli stresses that a scientific view of the material world comes out of the preceding archetypical view. The influence of Jung is evident here. Of the same archetypical origin is the Pauli's 'division and reduction of symmetry'. One day Heisenberg asked Pauli "why he laid so much stress on the doubling process", and Pauli "made the following reply: In the earlier physics of the atomic shell we had to rely exclusively on perceptual models taken from the arsenal of classical physics. Bohr's correspondence principle stressed the usefulness, however limited, of such models. But the mathematical description of what goes on in the atomic shell was always much more abstract than such models. In fact, it is quite possible to attribute quite different and mutually contradictory models, for example, the particle and the wave models, to the same real process. In the physics of elementary particles, however, all such models prove of no practical use at all, for that branch of science is even more abstract. If we wish to formulate the physical laws in this realm, we must therefore base ourselves on the properties of symmetry that nature herself has introduced here, or, to put it differently, on the symmetry operations (for instance, displacements and rotations) that open up nature's space. Now this forces us to ask why there are these symmetry operations and no others. I think that the concept of division or doubling will prove particularly useful here, because it helps to extend nature in what seems to be an unforced manner, and may thus introduce new symmetries. In the ideal case, we could imagine that all real symmetries have come about as a result of this kind of division" [29, p. 232].

In December of 1957, Pauli wrote to Heisenberg: "The picture keeps shifting all the time. Everything is in flux. Nothing for publication yet, but it's all bound to turn out magnificently. No one can tell just what marvels will appear. Wish me luck, I am learning to walk. [And then the quotation:] Reason begins again to speak, again the bloom of hope returns. The streams of life we fain would seek, ah, for life's source our spirit yearns. Greet the dawn of 1958 before sunrise. . . . Enough for today. This is powerful stuff. . . . **The cat is out of the bag**, and has shown its claws: division and symmetry reduction. I have gone out to meet it with my antisymmetry—I gave it fair play" [29, p. 234]. (Mind that in the English translation the German 'Pudel' had turned into the 'cat'; here is the German original: "Du wirst bemerkt haben, daß der **Das ist des Pudels Kern** fort ist. Er hat seinen Kern enthüllt, Zweiteilung und Symmetrieverminderung. Ich bin ihm da mit meiner Antisymmetrie entgegengekommen - ich gab ihm fair play - worauf er sanft entschwand...").

The epilogue of this story came a few months later. Early in 1958 Pauli had to leave for America, where he had lecture engagement. Heisenberg tried to stop Pauli: "I did not like the idea of this encounter between Wolfgang in his present mood of great exaltation and the sober American pragmatists, and tried to stop him from going. Unfortunately, his plans could no longer be changed. <…> Then we were divided by the Atlantic, and Wolfgang's letters came at greater and greater intervals. <…> Then, quite suddenly, he wrote me a somewhat brusque letter in which he informed me of his decision to withdraw from both the work and the publication. He added that he had informed the recipients of the preliminary draft that its contents no longer represented his present opinion... But this did not fully explain his behavior. I myself was only too aware of the difficulties, but we had often worked together in the dark, and as far as I myself was concerned such situations had always struck me as the most interesting" [29, p. 234-235].

In his memorials, Pais writes: "Pauli's first stop was New York. He had requested to be allowed to give a 'secret' seminar on his recent work with Heisenberg at Columbia University, by invitation only. Actually, he spoke in the overfilled large lecture hall in Pupin Laboratory. I was present and vividly recall my reaction: this was not the Pauli I had known for so many years. He spoke hesitantly. Afterward, a few people, including Niels Bohr and myself, gathered around him. Pauli said to Bohr: 'You may well think that all this is crazy.' To which Bohr replied: 'Yes, but unfortunately it is not crazy enough'" [38, p. 250].

Judge for yourself whether Pauli with his idea of bipartition had come so close to some dark force that it got involved and reduced his initially hopeful enthusiastic effort to nothing but disappointment in the end…:) To be serious, today we can interpret bipartition and symmetry reduction as the reduction of group symmetry occurring in the following manner: given a chain of nested groups $G \supset G_1 \supset G_2 \supset \cdots \supset G_k$ and an irreducible unitary representation $\mathfrak{P}$ of the highest group G in Hilbert space $\mathcal{H}$, we notice that the reduction G/G_1 of representation $\mathfrak{P}$ by the subgroup G_1 makes $\mathfrak{P}$ reducible and expandable into an orthogonal sum of irreducible representations $\mathfrak{P}_i^{(1)}$ of subgroup G_1. In turn, the reduction G_1/G_2 of group G_1 representation by subgroup G_2 expands representations $\mathfrak{P}_i^{(1)}$ into a sum of irreducible representations $\mathfrak{P}_{ij}^{(2)}$ of subgroup G_2, and so on. That is how symmetry flows down from a high symmetry of group G to the lower symmetries of its subgroups.

Q.: Can we look at duality as a criterium of being, i.e., to exist means to have one's opposite? To exist in space implies to have the top and bottom, the left and right, i.e., literally to consist of opposites. It is only zero that has no opposite, to be equal to zero is well expressed by the verb 'vanish'. By the way, it follows that a point cannot exist in space; is it right that some physical theories (e.g., twistor theory [40]) consider a point as a space of spinors?

A.: We have mentioned already the zero dimension of a point and the Urysohn--Menger dimension theory. It makes no sense to represent 3d space (or 4d spacetime) by a set of dots unless one takes into account the Hausdorff separation axiom. The doubling-up so stressed by Pauli is exactly the duality, i.e., the coexistence of two opposites. This idea can be found in Plato's Epinomis: "… a divine and marvelous thing it is to those who envisage it and reflect, how the whole of nature is impressed with species and class according to each analogy, as power and its opposite continually turn upon the double. Thus the first analogy is of the double in point of number, passing from one to two in order of counting, and that which is according to power is double; that which passes to the solid and tangible is likewise again double, having proceeded from one to eight" [39, p. 34].

Doubling (or duality) is a universal feature of matter. The spin of electron is a manifestation of duality rather than an internal property of a thing. Recall that electrons can be observed only having a certain direction of spin and not both of them simultaneously. Duality preexists in matter (substance), which endows its accidences (electrons) by a certain value of spin depending on the experimental situation (manifestation).

Q.: Interaction means an exchange by properties. To make an exchange, the objects should come to the 'neighboring points' in spacetime. In the absence of spacetime at subatomic level, how can we describe interactions?

A.: You described how it goes from the reductional viewpoint. It is based on the concept of field which carries the action from point to point and needs a continuum spacetime for that. It is a mechanical model where the particles of matter (fermionic fields) interact by means of the carrier particles (gauge fields). Recall that the Planck's law permits only discrete values of energy and, hence, the idea of continuum is out of place and even harmful since it makes things obscure and hard to deal with. Eliminating continuum at the basic level ($\sim l_0$) destroys also fields and generally the theories of field-aided interactions, which are built on mechanical models borrowed from classical physics (lagrangians, field equations, etc.). When elaborating his theory of spinor matter, Heisenberg could not entirely get rid of the field methods, which was the reason of failure.

The continuum field paradigm is inappropriate at the atomic scale and should give place to its opposite, based on the *action-at-a-distance* concept, as required by the holistic approach. How can we make it work? A brief answer is -- by implementing into the 'spectrum of matter' theory. The key point of the theory is that the states of matter are of emergent nature and thus cannot interact in a mechanical manner. The action-at-a-distance is not mechanical, it is realized at the substantial level. The states ω of the theory are given by the cyclic vectors of $\mathbb{K}$ -Hilbert space. These states are pure and separable and generate a separable state if their product makes a convex linear combination of pure states. Otherwise, the generated state is nonseparable (entangled) and corresponds to the interaction of states. This way of interacting has nothing to do with mechanical forces and fields and is obviously of a different kind.

Q.: Do you agree that continuum mathematics is of no use to describe quantum world? On one hand, it would be a relief since we need not keep trying to fit a square peg into a round hole, but on the other the perfectly elaborated integrodifferential machinery of conventional QFT appears to be out of play. Do you see any alternative?

A.: In his last paper in 1955 Einstein wrote: "One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory and must lead to an attempt to find a purely algebraic theory for the representation of reality. But nobody knows how to find the basis for such a theory" [41, p. 166].

Since 1955 physicists had tried numerous variants, and some of them look perspective; however, even a brief review of them would be a long talk, which is better to postpone until next time.

Thank you very much for the detailed answers and a most illuminative talk! We wish you new insights and look forward to our next meeting.

Interviewed and translated into English by Anna Sidorova-Biryukova.

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