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Is Sirius a triple star?

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Abstract. Sirius has been discovered as double more than 130 years ago. From the beginning of our century up to now, observational as well as physical and dynamical indications lead to the hypothesis of the existence of a third body in the system. In this paper, we present recent orbital analysis of the binary Sirius A–B which, helped by numerical simulation of triple systems, strengthens the idea for the triplicity of Sirius: a tiny star could revolve in about 6 years around Sirius A*. Finally, we discuss the possibility of direct detection for this suspected Sirius C.

Key words: stars: αCMa – celestial mechanics – binaries: general – low mass stars, brown dwarfs

1. Introduction

Sirius, "The Bright" of the ancients, forms, with its companion discovered a little more than 130 years ago, one of the most amazing double – and maybe triple – star.

Measured as soon as the ancient Egyptians established the relation between the Nile in spate and the first seeing of "Sothis" at dawn – the "heliacal rising" –, Sirius plays an important role in astronomy, for example in the discovery of stellar proper motions (see e.g. Lacaille 1764); in other respects, everybody knows that the evolutionary interrelations between the two stars is still an open question.

The proper motion of Sirius itself, well known since Halley's times, shows periodic variations. Bessel proposed in 1844 the hypothesis that these variations are due to an unseen companion. A theoretical orbit for the suspected double star was computed by Peters in 1851, Safford in 1861 and Auwers in 1862; in this latter year, Alvan Clark actually discovered the now well-known white dwarf Sirius B; the main star of the binary is therefore called Sirius A. So Sirius' companion is one of the first heavenly bodies the existence of which had been predicted through its gravitational effects, together with Neptune.

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* Tables 2 and 3, and the table in Appendix are available in electronic form at the CDS via *anonymous ftp 130.79.128.5*.

Since then, the 2 stars have revolved around each other more than 2 times; the current values of the orbital elements are those computed by Van den Bos in 1960 (Fig. 1). The angular distance between Sirius A and B varies from around 4″ at periastron, where the companion is lost in the very brightness of the most shining star in the Earth's sky, up to almost 12″ at aphelion.

As soon as 1894, irregularities were found in the motion of Sirius B, and lead to suspect the existence of a third body in the system. Moreover, observations, mainly in the 1920's, and reflexions on physics of Sirius A, came and supported this hypothesis. But this is still controversial, as we will see below (for more historical details about this section, see e.g. Allen 1963; Baize 1931, 1987; Benest 1987).

1.1. The controversial Sirius C

A tiny star ($m_v \approx 12$) has been observed about twenty times between 1920 and 1930. If there was a real object and not a "phantom" – the observers themselves were sometimes in doubt –, an orbit of around 2 years could roughly agree. However, we will see that this period does not fit with the results of the orbital analysis.

On the other hand, an analysis of the radial velocity of Sirius A between 1899 and 1926 led Voronov (1933, 1934a,b) to the hypothesis of the duplicity of Sirius A, with an orbital period of 4.5 years. Heintze (1968) also suspected such a duplicity, from the spectrum of this star, and concluded in favour of a relatively close companion of Sirius A. However, Lindenblad (1973), after photographic measurements over 6.8 years, did not find any significant perturbation. Moreover, Gatewood & Gatewood (1978) analyzed 60 years of observations with the Allegheny and Yerkes refractors, and concluded that nothing supports the hypothesis of a third body. Nevertheless, recent discussions about a possible change of color of Sirius during historic times (Schlossen & Bergmann 1985; Tang 1986; van't Veer & Durand 1988; Gry & Bonnet-Bidaud 1990) relaunch the debate: if the phenomenon is real, one of the possible explanations is the existence of a third body; Bonnet-Bidaud & Gry (1991) have observed the vicinity of Sirius and proposed several faint stars ($m_v \geq 17$) as candidates, but they are all so far from

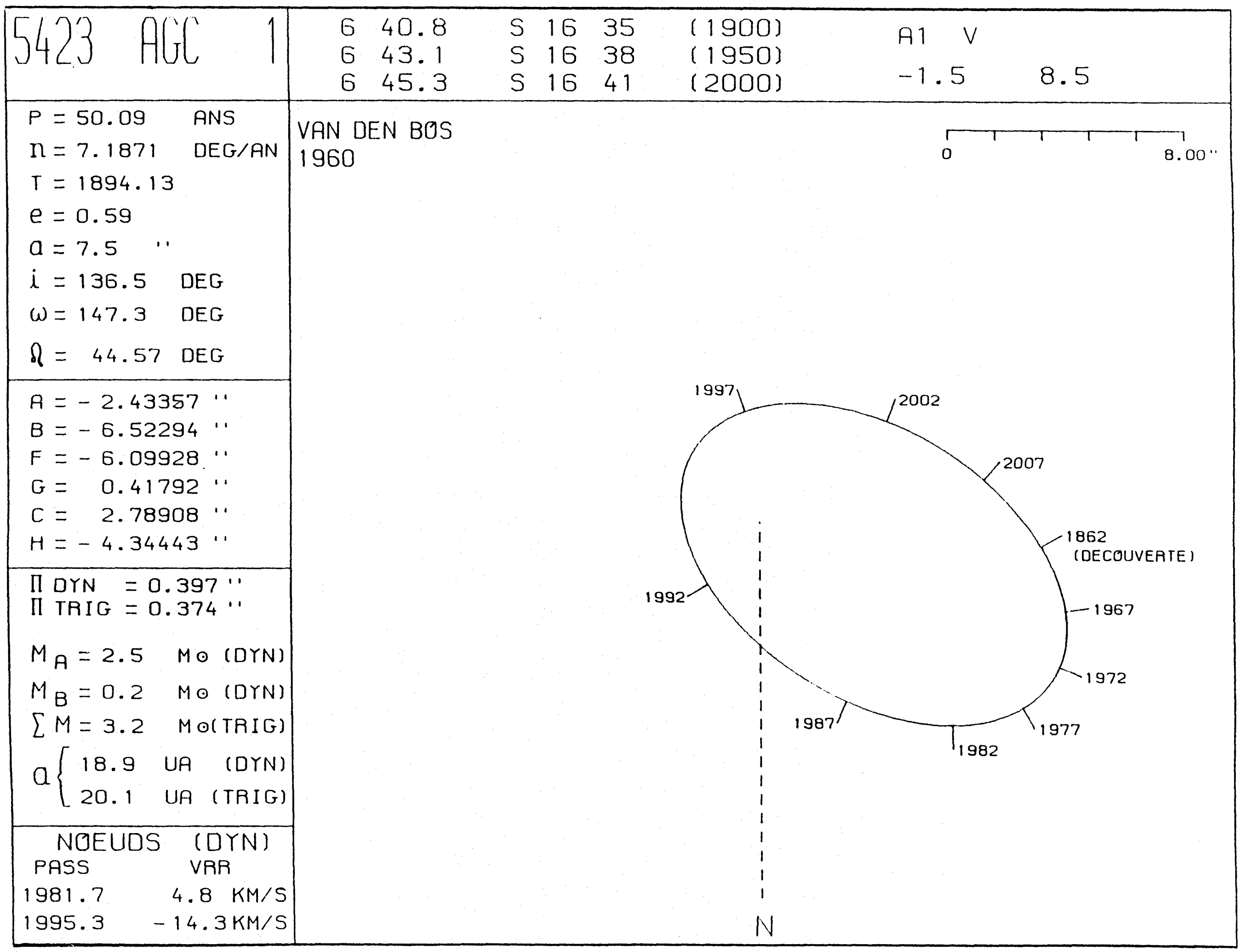

Fig. 1. Apparent orbit of Sirius B around Sirius A (from Couteau & Morel 1973)

Sirius B that they could actually be only remote companions of the system (see also Bonnet-Bidaud & Gry 1992).

Now, let us talk more about the orbital analysis. Our goal is to extract from the "O–C" (the difference between the observations and the computed theoretical orbit) the period of the suspected perturbation, which could correspond to the orbital period of the potential third body. This period may be searched through a study of variations of the areal velocity; Zagar in 1932 used this method and found a period of 6.3 years, for a perturbing body around Sirius B; but we must note that this method requires several hypotheses so that the result depends on the chosen procedure (see Volet 1932). More simply, and more rigorously, the period of the perturbation can be obtained by a least square algorithm, which Volet (1932) solved analytically. He proposed then the value of 6.4 years around Sirius B, but possibly also around Sirius A; more recently, one of us used an analogous algorithm for numerical computation and obtained a value around 6 years (this will be developed in Sect. 2). Finally, another possibility is simply to apply a Fourier analysis to detect the period of perturbation; Walbaum & Duvent (1983) found with this method a value one more time around 6 years. We have therefore three independent methods giving rather close values; we think then that this is a rather strong indication in favour of the reality of the perturbation.

But what could be the cause (or the causes) of this perturbation? Let us limit ourselves to the hypothesis of the existence of a third body in the Sirius system, thereafter called Sirius C. Next question is: does this Sirius C revolve around Sirius A, around Sirius B, or far around the binary Sirius A–B? This cannot be actually determined by the orbital analysis (too many parameters). Fortunately, celestial mechanics can help us now, either by analytic study or by numerical or semi-numerical simulation: i.e. do stable orbits exist around Sirius A and/or Sirius B with a period of 6 years? do stable orbits exist far around Sirius A–B which could perturb (for example through a resonance) the orbits of Sirius A and B around each other, again with a period of 6 years? The latter has been explored by, e.g., Donnison & Williams (1978; see also, for more general cases, Dvorak et al. 1989; Kubala et al. 1993).

One of us, during a long-term systematic study of the stability of third body's orbits in multiple systems, has previously shown that, generally speaking, many such stable orbits exist around one of the two components in a binary (see e.g. Benest 1988b, 1993), and that, more precisely, such stable orbits exist around Sirius A as well as around Sirius B, although stable orbits with period about 6 years exist only around Sirius A (Benest 1989); this will be developed in Sect. 3.

Finally, in Sect. 4, we will discuss our results and propose an observational challenge: the direct detection of the suspected Sirius C.

Table 1. New orbital elements for Sirius, compared with the classical elements computed by Van den Bos in 1960

	New elements	Van den Bos' elements
P = period (years)	50.052	50.09
Epoch of periastron	1894.164	1894.13
e	0.5923	0.59
a (arcsec)	7.501	7.5
i (°)	136.62	136.5
ω (°)	148.07	147.3
Ω (°)	44.86	44.57

2. The orbital analysis

Several methods can be used to extract, from the "O–C" (i.e. the difference between the observed and computed positions), the period of a suspected perturbation.

In a previous study (Walbaum & Duvent 1983), a Fourier transform was processed on the O–C in cartesian coordinates:

– The data samples were obtained through computing the mean of selected observations for each year.

– The seven orbital elements (period P, semi-major axis a, eccentricity e, inclination i, argument of periastron ω, longitude of node Ω and epoch of periastron) were optimized through a mean square O–C minimization.

– Computation was done in the Sirius A–B orbital plane.

We obtained two peaks in the amplitude perturbation curve:

I. Period = 6.05 years, amplitude = 0.095 arcsec, estimated probability of existence = 90%;

II. Period = 6.70 years, amplitude = 0.085 arcsec, estimated probability of existence = 10%.

More recently, we decided to enhance the method up to obtain more accuracy, by using all individual observations of the binary Sirius, and by taking care of the quality of measurements; this was possible using mean square error methods, and weighting measures. Data used for our calculations are all observations made from the discovery of Sirius B (1862) up to 1979 (see Appendix, available in electronic form).

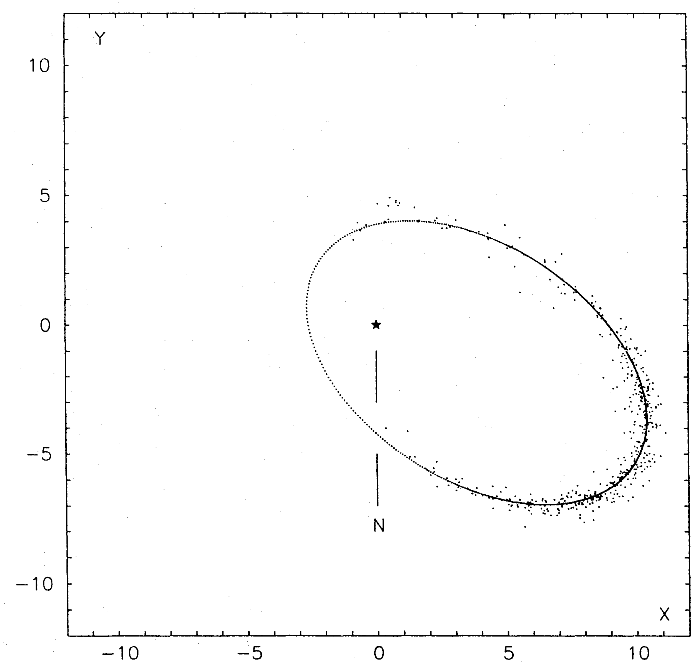

Fig. 2. Fit of the new orbital elements (in the apparent plane); coordinates are in arcsec; see Appendix for details on the observations

2.1. Orbital elements optimization

First, we have to optimize the seven elements of the A–B orbit, because if we don't we could find harmonics of main period A–B in O–C frequency analysis. By iteration, the value of each element minimizing the mean square O–C in A–B real orbit plane is determined. Table 1 presents our result and compare them with the classical elements computed by Van den Bos in 1960. The agreement between the new orbital elements and the observed values is shown in Fig. 2.

2.2. Mean square Fourier estimation

Now, we analyse O–C functions in real plane. The O–C are calculated in cartesian coordinates with x-axis chosen to be the intersection of apparent orbit plane and real orbit plane, i.e. the line of nodes.

Each O–C function is processed as follows :
Let

$$F(t) = \sum_{k=1}^{q} A_k \cos(k\Omega t) + \sum_{k=1}^{q} B_k \sin(k\Omega t)$$

where $q < P/2$ (following the Nyquist criterion –here we have chosen $q = 25$–), $\Omega = 2\pi/P$, A_k and B_k are the Fourier coefficients; and let P_i be the weight of the value $F_i = F(t_i)$ for $i = 1, n$ where n is the number of observations; we have chosen $P_i = \sqrt{n_i}$ where n_i is the number of nights for every observational set. Then, if we define the vectors

$$\boldsymbol{F} = \begin{pmatrix} F_1 \\ F_2 \\ \dots \\ F_n \end{pmatrix}, \boldsymbol{V}_k = \begin{pmatrix} \cos(k\Omega t_1) \\ \cos(k\Omega t_2) \\ \dots \\ \cos(k\Omega t_n) \end{pmatrix}, \boldsymbol{V}_{q+k} = \begin{pmatrix} \sin(k\Omega t_1) \\ \sin(k\Omega t_2) \\ \dots \\ \sin(k\Omega t_n) \end{pmatrix}$$

and the matrix $n \times 2q$

$$\boldsymbol{M} = (\boldsymbol{V}_1, \boldsymbol{V}_2, \dots \boldsymbol{V}_q, \boldsymbol{V}_{q+1}, \boldsymbol{V}_{q+2}, \dots \boldsymbol{V}_{2q})$$

we can write

$$F = M\,S, \tag{1}$$

where

$$S = \begin{pmatrix} A_1 \\ A_2 \\ \dots \\ A_q \\ B_1 \\ B_2 \\ \dots \\ B_q \end{pmatrix}$$

is the unknown vector to find.

The matrix M is not a square matrix, so we must solve Eq. (1) using the mean square error minimization criterion allowing measurements weighting: $\varepsilon_1 = \|P\,(F - M\,S)\|^2$, where P is the n-dimensional diagonal matrix with the P_i's as elements of the diagonal, is the quantity to be minimized.

Nevertheless, as observation errors are very large, it is necessary to add one more criterion to avoid values out of range in S, i.e. the minimization of $\varepsilon_2 = \sigma^2\,\|IS\|^2$, where I is the identity matrix and σ a weighting parameter; σ is determined experimentally to obtain coherent values in S, and is thus proportional to the standard deviation of the measures.

To conclude, we have to minimize :

$$\varepsilon = \|P\,(F - MS)\|^2 + \sigma^2\,\|IS\|^2 \tag{2}$$

Then, we get :

$$\left((PM)^t\,(PM) + \sigma^2\,I^t\,I\right)\,.\,S = (PM)^t\,.\,(PF) \qquad (3),$$

whose solution is:

$$S = \left((M^t\,P^2\,M) + \sigma^2\,I^t\,I\right)^{-1}\,.\,M^t\,P^2\,F \tag{4}$$

Using Eq. (4), we analyse O–C$_x$ and O–C$_y$. For each, we obtain the amplitude function of the frequency expressed in year^{-1}.

If the perturbation is real, we must find a peak at the same frequency for O–C$_X$ and O–C$_Y$. To evaluate the correlation of the peak, we calculate the geometrical mean of amplitudes on X-axis and Y-axis for each frequency. Thus, we can write the mean of the Fourier transforms of O–C$_X$ and O–C$_Y$ as $A_{xy}(f) = \sqrt{A_x(f) \times A_y(f)}$, where $A_x(f)$ (resp. $A_y(f)$) is the amplitude of the Fourier transform of O–C$_X$ (resp. O–C$_Y$) for the frequency f.

We have determined experimentally that the best fit is obtained for $\sigma = 10$. Then a maximum of amplitude appears for about 6.25 years (Table 2, available in electronic form, and Fig. 3), which agrees well with our previous results.

2.3. Mean square sine function estimation

However, a disadvantage of the latter method is the rather poor resolution. Therefore, we decided to also search a sine function matching with the measures, using a mean square error minimization criterion.

The process is thus to find the best phase of a sine function for which the amplitude is calculated by linear equations. If we write the sine function as

$$\sin(\omega t + \varphi) = \sin\omega t\cos\varphi + \cos\omega t\sin\varphi = A\cos\omega t + B\sin\omega t$$

we define

$$\varepsilon_1 = \sum_{i=1}^{n}(A_X\cos\omega t + B_X\sin\omega t - \mathrm{O\text{–}C}_{X\,i})^2$$

and

$$\varepsilon_2 = \sum_{i=1}^{n}(A_Y\cos\omega t + B_Y\sin\omega t - \mathrm{O\text{–}C}_{Y\,i})^2$$

where n is the number of observations. Then, for a sample of values of ω such that $\omega_j = k^j\omega_0$ with $\omega_0 = 2\pi/P$ and k adjusted for an appropriate sampling, the A_X, B_X, A_Y and B_Y are determined through

$$\frac{\partial\varepsilon_1}{\partial A_X} = 0\,,\ \frac{\partial\varepsilon_1}{\partial B_X} = 0\,,\ \frac{\partial\varepsilon_2}{\partial A_Y} = 0\,,\ \frac{\partial\varepsilon_2}{\partial B_Y} = 0\,;$$

the final quantities considered are the two amplitudes on O–C$_x$ and on O–C$_y$, $\mathscr{A}_X = \sqrt{A_X^2 + B_X^2}$ and $\mathscr{A}_Y = \sqrt{A_Y^2 + B_Y^2}$, together with their mean $\mathscr{A}_{XY} = \sqrt{\mathscr{A}_X\,\mathscr{A}_Y}$.

The analysis is made both for all data and for means through a 1 year sampling step; in the latter case, the weighting secures the analysis efficiency, although the poorer sampling rate gives a lesser resolution.

In Table 3 (where $k = 1.0045$; available in electronic form) and Fig. 4, the perturbation peak appears clearly on $\mathscr{A}_{XY}$, both for all data and for the 1 year sampling, for a period of 6.3 years and an amplitude of 0.056″.

2.4. Concluding the orbital analysis

Process enhancements, more data used and two new methods confirm the previous studies, that a 6.3 years period perturbation probably exists in Sirius A–B orbit. More accuracy shows that the perturbation amplitude is 0.055″; as we know Sirius' parallax, A mass and B mass, we can estimate C mass and distance from its primary, which, as shown in the next section, is very likely Sirius A.

3. The numerical experiments

The conclusion of the preceding section is therefore a fairly high probability of existence for a perturbation of period around 6 years acting on the orbit of the binary Sirius. As said in the introduction, we suppose that this perturbation is due to the presence of a third star in the system, revolving with a 6 years period

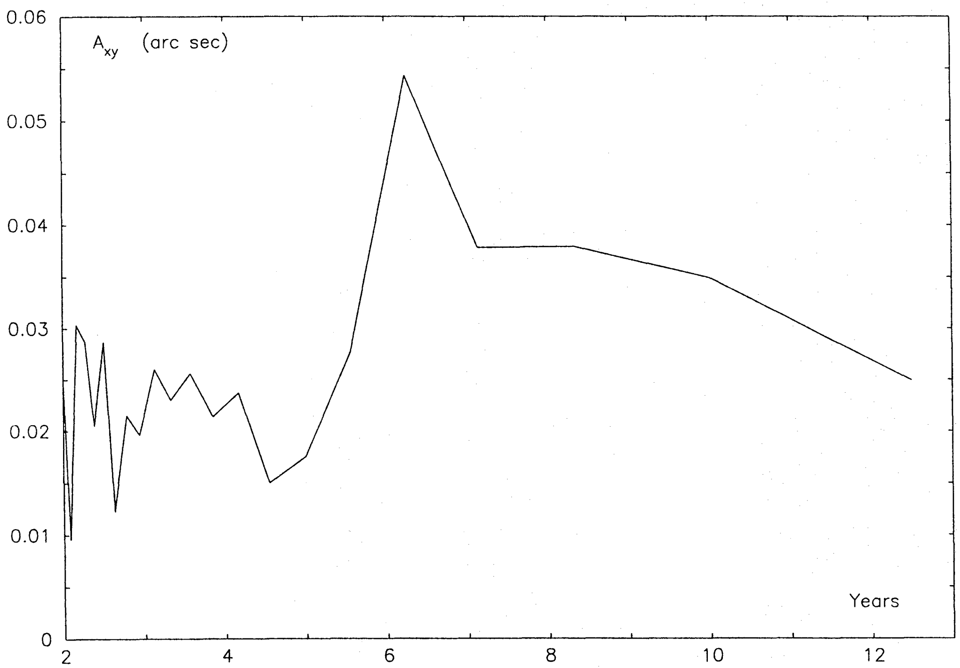

Fig. 3. Amplitude of A_{xy} in the mean square Fourier estimation, showing the peak at about 6.25 years

either around Sirius A or around Sirius B. Unfortunately, the great number of parameters of the problem makes the solution not unique, i.e. the mass of Sirius C is not well determined, inducing large uncertainties on the orbital elements of this suspected companion; moreover, we can't determine around which star revolves Sirius C (around Sirius A or around Sirius B?).

Numerical experiments may then help us to determine which orbital configurations are possible or not. For this purpose, a classical way is to use a three-body model, in which we compute the motion of three point masses subject only to gravitational forces; their motion depends then of their masses and their initial coordinates and velocities, i.e. 21 parameters in the most general three-dimensional case, and 15 in the planar case (where the three bodies move in the same plane)! Of course we must use this model when the three masses are of the same order. Nevertheless, the mass of Sirius C is probably low, otherwise the perturbations would destroy the binary very rapidly; we may roughly estimate the maximum value for $M_{\rm C}$ to be around 0.05 $M_{\odot}$ (Sirius C would then be at best a red dwarf M5 with $m_v \geq 12$), which is much less than the masses of Sirius A and B, resp. 2.14 and 1.05 $M_{\odot}$ (from Gatewood & Gatewood 1978). We may therefore use a particular approximate case of the three-body model, the restricted problem (see Szebehely 1967).

3.1. The restricted three-body model

In the restricted three-body problem, we suppose that the mass of one body is so small with respect to the two others (the "heavy" binary) that it does not perturb their keplerian motion around each other; of course, it does not allow to study the perturbation on the binary's orbit, but it is a quite good model to study in a first stage the possible motions of the tiny body, here Sirius C, around its heavy primary, here Sirius A or Sirius B, particularly to know if stable 6-year orbits exist; the general three-body model can be used afterwards to extend and confirm (or invalidate) the results found with the restricted problem. An advantage of the restricted model is that the number of parameters falls down to 8: the reduced mass of the primary of Sirius C $\mu = M_{\rm P}/M_{\rm B}$ (where $M_{\rm P}$ is the mass of the primary and $M_{\rm B}$ is the total mass of the binary), the eccentricity e of the orbit of the binary, and the initial coordinates and velocity of Sirius C. Moreover, we may limit ourselves to the planar case, i.e. the tiny body moves in the same plane than the orbital plane of the binary; this is not a so strong limitation, as Harrington (1972) has shown that the stability of such systems does not depend in general on the inclination i between the two orbital planes, apart for i near 90°; the number of parameters now falls down to 6. Finally, the equations are classically written in a rotating-pulsating frame where the two heavy bodies are at rest on the X-axis, and the primary of the tiny body is at the origin (the other massive body is at $X = -1$; the distance unit is the instantaneous separation between the two massive bodies, and the time variable is the true anomaly of the binary's orbital motion); it has then been shown that we may limit the initial conditions to a departure of the tiny body perpendicularly from the positive part of the X-axis, without restricting the generality of the search, so that initial coordinates and velocity restrict to $X_0 > 0$ and $\dot{Y}_0 = V_0$ ($\dot{X}_0 = 0, Y_0 = 0$); therefore, for each set (μ, e) of parameters, we can easily visualize in the half-plane $(X_0 > 0, V_0)$ the region corresponding to stable orbits (for more details, see e.g. Benest 1971, 1978). In our computation, we consider that an orbit can be called stable if there has been neither escape nor collision

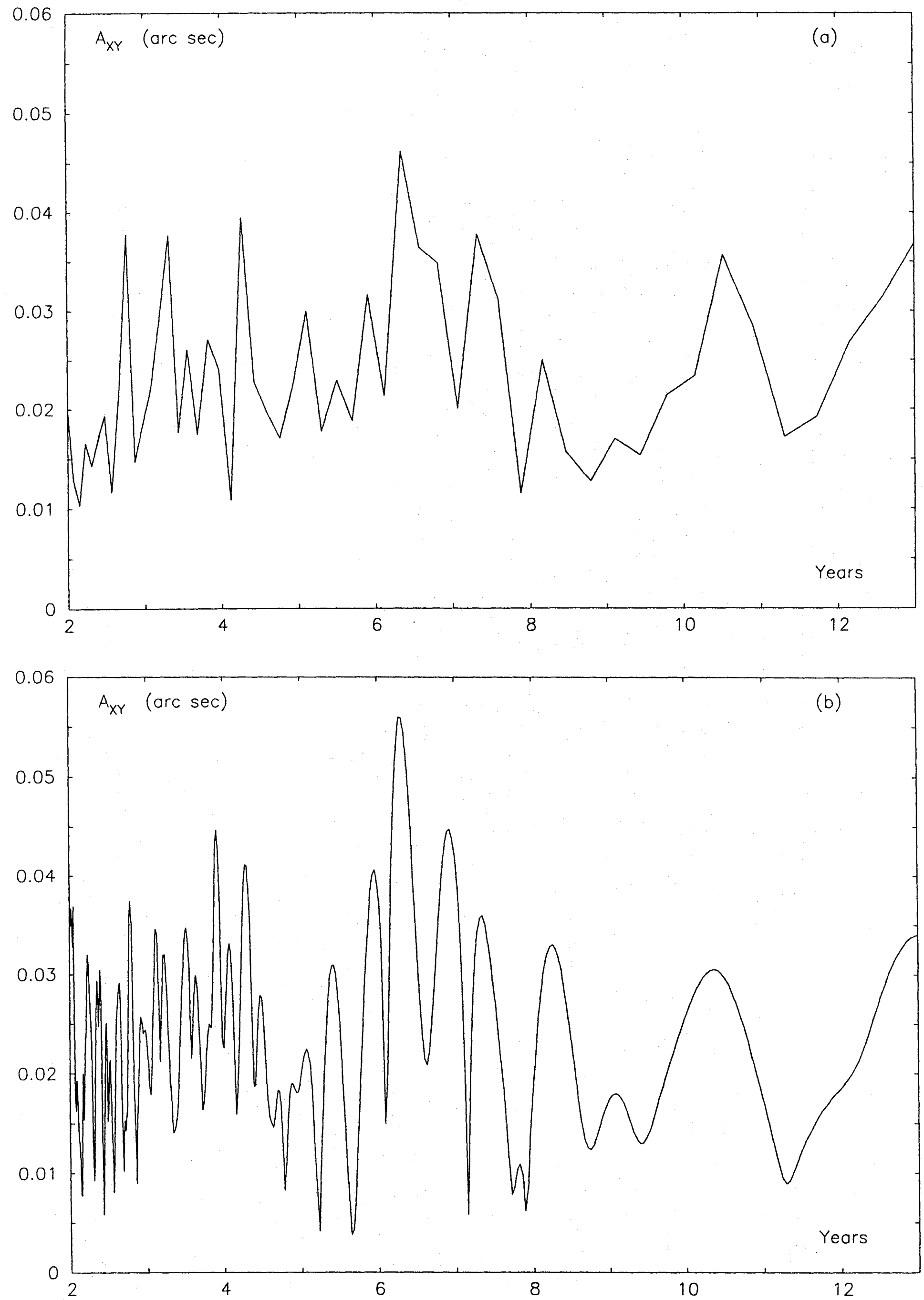

Fig. 4a and b. Amplitude of $\mathscr{A}_{XY}$ in the mean square sine function estimation, showing the peak at about 6.3 years: **a** for all data, **b** for a 1-year sampling

during 100 revolutions of the binary (about 5000 years in the case of Sirius); it could seem short compared with astronomical ages, but previous numerical experiments have shown that in general orbits reveal their unstable character before this limit, and that longer integrations product only very little shift of the boundaries of the stability region (Benest 1988b).

During the 1970's, the circular (e = 0) restricted problem has been systematically explored for every value of μ between 0 and 1, and stable orbits were found to exist as far from their primary as about the separation of the binary (Benest 1974, 1975, 1976). More recently, the study of the elliptic case ($e > 0$) has been started on, beginning with values of μ and e corresponding to actual binaries, first α Centauri (Benest 1988a); the Sirius system has then been investigated ($\mu = 1/3$ and $2/3$; $e = 0.592$). A subproduct of this latter exploration was the search for stable orbits of period around 6 years: such orbits have been shown to exist around Sirius A, near the inner boundary of the stability region, but not around Sirius B (Benest 1989).

3.2. The general three-body model

The next question, as stated above, is: although 6-year stable orbits do not at all exist around Sirius B in the restricted problem (the longest mean periods are less than 5 years), do such orbits exist in the general problem, and, if yes, for which masses of the three bodies? For this purpose, the dynamical model is now the plane general problem of three bodies, i.e. with three finite masses; the equations, regularized using Waldvogel's method, are integrated by the classical Bulirsch-Stoer algorithm (for more details, see Hénon 1974).

Table 4. Stability of Sirius C, depending on M_C (mass of Sirius C, in Solar Mass) and a_0 (the initial semi-major axis of the orbit of Sirius C around Sirius B, in AU); plane general three-body model; S = stable orbit, followed (inside parenthesis) by the mean orbital period of Sirius C around Sirius B, in years; U = unstable orbit

a_0 / M_C	1	2	2.5	3	3.5	4
0.038	S (0.5)	S (1.6)	U	U	U	U
0.02	S (0.6)	S (2.1)	S (2.8)	U	U	U
0.01	S (0.8)	S (2.5)	S (3)	U	U	U
0.001	S (0.95)	S (2.6)	U	U	U	U
$3 \cdot 10^{-5}$	S (0.95)	S (2.8)	U	U	U	U

Within this frame, we explore the stability of the orbit of a Sirius C orbiting around Sirius B, varying its mass and its initial conditions. Table 4 shows that not any stable orbit exists for a period more than 3 years, thus confirming the results from the restricted three-body model.

3.3. Concluding the numerical experiments

Numerical simulations of the possible triple system Sirius A–B–C indicate that no stable orbit for Sirius C exists around Sirius B with a period greater than 4 years, both using the restricted and the general three-body models. Fortunately, stable orbits with 6-year period exist for Sirius C around Sirius A.

4. Discussion and prospects

In this study we have supposed that the 6-year perturbation in the orbit of Sirius A–B is due to the presence of a third body in the system, i.e. Sirius C. Dynamical and astrophysical considerations lead to a very low mass for this suspected companion: a maximum of 0.05 $M_\odot$, i.e. a M5 to M9 star of absolute magnitude 15 to 20. The visual magnitude of Sirius C could then be 5 to 10 more than that of the white dwarf Sirius B (m_v = 8.5), itself 10 more than that of Sirius A ($m_v = -1.5$), therefore very difficult to observe. Moreover, the angular distance between Sirius A and B varies from 4″ at periastron to 12″ at apoastron; as the two bodies are nowadays near periastron, even Sirius B is not so easy to see in the very light of its bright primary.

The 6-year orbits found numerically are more or less elliptical (see Fig. 4 in Benest 1989), so that they can extend up to distance of Sirius A of the order of 2/3 to 3/4 of the periastron distance of Sirius A–B, i.e. up to 2″ to 3″ of Sirius A; therefore, as we cannot tell where Sirius C is on its orbit, we can just say that this body is to be searched at distances of Sirius A from 0″ to 3″ ; of course, as it may be nowadays hidden behind Sirius A, one negative observation does not imply that the object does not exist, for this conclusion could be stated only if no detection at all occurs after regular observations during at least half a period of Sirius C, i.e. 3 years.

But we are not limited to visual light. Considering indeed the respective magnitudes of the sources and their spectral types, other wavelength are more appropriate: for example the infrared K band, in which the magnitude of Sirius C could be less than 10, thus reducing the difference with Sirius A ($K = -1.4$). And this K band is available on many mountain observatories. Moreover, new technics – for example: adaptive optics – allow to observe faint stars with good signal/noise ratio. We know that Sirius A is so bright that it rapidly saturates any sensitive infrared camera. Fortunately, the recent progress in stellar coronagraphs let us hope that this technic, together with the use of adaptive optics, could finally allow to find Sirius C, if it exists, or at least give a maximum value for its luminosity, and hence for its mass.

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