Numerical and Experimental Results for a Novel Propulsion Technology Requiring no On-Board Propellant

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A proof-of-concept (POC) prototype of a new thruster technology was experimentally and numerically tested. The technology consists of an axially-asymmetric resonating cavity that is operated with a TM_{010} resonating electromagnetic (EM) wave. This new propulsion technology requires no on-board propellant to generate thrust. Numerical method analysis of the POC cavity shows that a differential in axially-directed Lorentz forces is exerted by the contained EM wave on the cavity when operated in the TM_{010} mode. Experimental results also show force generation by the POC cavity when powered with phase-locked RF energy. The numerical and experimental results from the POC tests are discussed.

Nomenclature

All units discussed in this paper are in the MKSA system, unless otherwise specified.

I. Introduction

Electromagnetic (EM) waves carry energy and momentum. In free space, the equation for energy transport of an EM wave is:

$$S = \frac{1}{\mu_0} (E \times B)$$

where:
S is the Poynting vector
\(\mu_0\) is the permeability constant
E is the electric field of the EM wave
B is the magnetic field of the EM wave

EM waves also carry momentum. In free space, momentum of an EM wave is related to the Poynting vector of an EM wave:

$$\rho_{EM} = \mu_0 \varepsilon_0 \int Sd\tau$$

where:
\(\rho_{EM}\) is the EM field momentum
\(S\) is the Poynting Vector of the EM wave
\(\mu_0\) is the permeability constant
\(\varepsilon_0\) is the permittivity constant

When EM waves pass through material media, the value for EM momentum differs from the free-space momentum of the wave. In 1908, Hermann Minkowski proposed a momentum density for EM waves passing through material media. In his work, Minkowski derives that the momentum of an EM wave increases within a material media proportional to the index of refraction of the material. In response to the work of Minkowski, Max

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Abraham published an alternate form for the momentum of an EM wave in material media. Abraham proposed that the momentum of a photon decreases proportionally to the index of refraction of the material media.

Both the Minkowski and Abraham expressions for EM momentum in media agree that EM momentum in media is different from the free-space momentum of the EM wave. Following Minkowski’s work, various techniques have been proposed for modulating EM wave momentum while the wave passes through material media. The purpose of this momentum modulation is to create a force on a device and produce thrust based on the difference in momentum values for the EM wave in material media vs. the free-space momentum of the wave. As of the date of this publication, no conclusive work confirming momentum-modulation effects in media have been reported in the literature. A more complete analysis of the various proposed techniques for EM momentum modulation in media is reported in LaPointe, and Robertson.

Additional techniques have been proposed which use electromagnetic field interactions with material media to generate a net force on the material media. Woodward-Effect thrusters are based on the interaction of electric and magnetic fields with a material medium. The EM-matter interactions induce mass fluctuations which are used to generate thrust. Various tests and descriptions of these devices are documented in the literature.

The device described in this paper is designed to generate thrust based on interactions of EM waves with a material medium. The device uses asymmetric interactions of the electric and magnetic fields of a resonating EM wave interacting with an axially-asymmetric resonant cavity to generate a net, time-averaged Lorentz force on the system. The present device generates thrust by a method previously unreported in the literature.

In order to demonstrate the technique of net Lorentz force generation, a superconducting radio frequency (SRF) proof-of-concept (POC) resonating cavity and experimental apparatus was built and tested. SRF resonating cavities are routinely used in linear-accelerator (LINAC) applications and other applications. However, the design of the POC cavity differs from typical LINAC SRFs and is new to the SRF cavity field. Instead of the traditional axially-symmetric, elliptical-cavity geometries currently used in LINAC applications, the POC cavity design includes axially-asymmetric features. When the cavity is operated in the TM010 mode, these axially-asymmetric features cause a contained EM wave to induce a net, time-averaged Lorentz force on the cavity.

II. Proof-of-Concept Experiment Design

The POC SRF cavity prototype is comprised of the axially-asymmetric resonating cavity depicted in Figures 1 and 2. Dimensions are in centimeters. The POC SRF cavity is a pillbox-type cavity with a TM010 frequency of approximately 1047.34 MHz. Both the top plate and the bottom plate of the POC cavity are made from the same material.

![Figure 1. Top plate of the POC cavity](image1)

![Figure 2. Bottom plate of the POC cavity](image2)

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solid cylinder of 250 RRR Niobium. The plates were fabricated on a CNC milling machine. The bottom plate of the
cavity has 72 identical slots. Each slot on the bottom plate has a mill-cut into both slot walls at approximately 1 mm
below the primary plane of the bottom plate. The top and bottom plates were electron-beam welded along the cavity
equator. The top plate has 2 signal ports. The welded cavity was treated with 2 standard 3-acid chemical polishing
 treatments prior to the experimental tests.11

The z-axis is collinear to the central axis of the POC cavity. The positive z direction points from the bottom plate
towards the top plate of the cavity.

Figure 3 depicts the POC cavity with attached signal cables and vacuum piping. The design of the vacuum piping
allows for adjustments to the position of the power cable. The adjustable power coupling is required to allow for
rapid burn-through of multipacting barriers, and to allow for positioning of the power cable at or near unity coupling
during experimental runs. The signal pickup cable is fixed in position. The power cable and signal pickup cable are
used in a standard phase-lock-loop arrangement to power the cavity with up to 30 watts of power forward at the
power input port.12

Figure 3. Assembled POC cavity with signal cables
and vacuum piping

Figure 4. Experimental apparatus with POC cavity

The POC cavity and attached vacuum tubing are supported only by the central vacuum pipe depicted in Figure 4.
The central vacuum pipe is attached to the support arm which is also depicted in Figure 4. During experimental runs,
the cavity, attached vacuum tubes, and support arm are supported by 2 Cooper Instruments LFS 210 load cells. The
removable bellows of Figure 4 is removed from the experimental apparatus during testing. The two signal cables
that are attached to the cavity are looped and supported above the experimental apparatus so that negligible vertical
support is provided by the cables to the experimental apparatus. During test runs, the cavity, vacuum tubing, and
support arm are supported only by the 2 load cells beneath the support arm and by buoyancy of the cavity in the
liquid-helium bath. Signals from the two load cells are sent to a summing unit. The output of the summing unit is
sent through a Cooper Instruments DCM 495 amplifier. The DCM amplifier output is routed through an Agilent
34110A 6-digit multimeter. The multimeter sends real-time signal output to a computer running Microsoft Excel and
the output of the load cells is charted and recorded in real time. During experiments, the Excel file is projected
through an LCD projector onto a screen. All experimental runs, including the real-time output of the load-cell
circuit, are recorded on HD video.

Prior to test runs, the helium vessel depicted in Figure 4 is vacuum sealed and pressure over the liquid helium is
reduced to 50 Torr, reducing the liquid helium temperature to 2.3 K. The helium vacuum pump-down to 50 Torr
requires approximately 30 minutes to complete. Helium gas is then fed into the helium vessel to bring the pressure

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in the vessel up to atmospheric pressure. The vacuum seal on the helium vessel is then broken, and the pressure above the liquid helium remains at atmospheric pressure during test runs. During test runs on the cavity the liquid helium bath remained below its atmospheric boiling temperature. The helium pump-down procedure eliminated boiling helium buoyancy beneath the cavity as a potential cause of false-positive experimental results.

Liquid helium temperature was not reduced below 2.3 K in order to prevent helium from reaching a superfluid state. On prior tests using superfluid helium, vacuum pressure inside the cavity increased as the superfluid helium migrated through the vacuum fittings of the apparatus.

III. Operating Principle

A. Lorentz Forces

During operation of an SRF cavity with a contained resonating EM wave, Lorentz force pressures are exerted on the walls of the SRF cavity. The Lorentz forces exerted on the cavity walls can distort the cavity shape and cause Lorentz force detuning of the cavity.\textsuperscript{11} Phase-lock-loop designs adjust the frequency of RF input power to accommodate frequency changes induced on the cavity by Lorentz force detuning effects.

In areas inside a resonating cavity where magnetic fields predominate, Lorentz forces are primarily generated by AC electric currents on the cavity walls interacting with the magnetic fields of the EM wave. The direction of the Lorentz force vector is perpendicular to both the current direction and the magnetic field direction, and is parallel to the surface-normal vector. The direction of the magnetic-field-induced Lorentz force vector is perpendicular to the cavity wall and points into the cavity wall from the EM field. In areas of a resonating cavity where electric fields predominate, Lorentz forces are primarily generated by surplus electric charges on the cavity walls interacting with the electric fields of the wave. The direction of the electric-field-induced Lorentz force vector is perpendicular to the cavity wall and points into the vacuum space inside the cavity.

The POC cavity relies on a differential in radiation pressure exerted by a $\text{TM}_{010}$ wave on the walls of the cavity to create a time averaged net Lorentz force. The radiation pressure of an EM wave interacting with media can be described by both the Lorentz force law, and by the Maxwell Stress Tensor (MST). The MST is derived from the Lorentz force equation and Maxwell's equations.\textsuperscript{1} The Lorentz force law is more practical for numerical analysis.\textsuperscript{13}

The Lorentz force law:

$$ F = \frac{d}{dt} \rho = q(E + v \times B) $$

(3)

where:

$\rho$ is the particle momentum
$q$ is the particle charge
$E$ is the electric field acting on the particle
$B$ is the magnetic field acting on the particle
$v$ is the particle velocity

The Lorentz force acting on a reflective surface can be treated as two separate forces: The electric field force which acts on electric charges, and the magnetic field force that acts on electric currents.\textsuperscript{13}

Within a resonating EM wave, energy oscillates between two states. At one phase of the wave cycle, 100% of the wave energy is present in the E-field. After a phase shift of 90 degrees, the wave energy is 100% present in the magnetic field. The electric and magnetic fields of a resonating wave are also out of phase with respect to location. Within a pillbox-type cavity operated in the $\text{TM}_{010}$ mode, the wave energy oscillates from concentration in the center of the cavity as electric field energy to concentration as magnetic field energy near the equatorial walls of the cavity. The distribution of field energy of the resonating wave is described with Bessel functions.\textsuperscript{14}

The total Lorentz force acting on a wall segment of a resonating cavity is the surface integral of the electric and magnetic force components:

$$ \vec{F} = \frac{1}{2} \int_S \mu_0 H_{max}^2 - \varepsilon_0 E_{max}^2 \, da $$

(4)

where:

$\vec{F}$ is the time-averaged net Lorentz force on the cavity over a complete wave cycle of the resonant wave.
$H_{max}$ is the maximum magnetic field during the wave cycle (A/m)
$E_{max}$ is the maximum electric field during the wave cycle (V/m)
Equation 4 is the complete time-averaged force experienced at the wall of the cavity. Any instantaneous shear forces experienced at the wall are cancelled when averaged over 1 wave cycle.

B. Lorentz Forces in the POC Cavity

Within the POC cavity, slots are located on the bottom plate (Figure 2) of the cavity. These slots are located in areas of high magnetic field and low electric field when the cavity is operated in the TM_{001} mode. The slots create geometric asymmetry in areas of high magnetic field. In the areas of the cavity that have high electric fields (near the axial center of the cavity), the cavity is geometrically symmetric.

During the wave cycle, AC current exists on the vertical surfaces (parallel to the z-axis) of the slots of the bottom plate and magnetic-field-induced Lorentz forces are generated on these surfaces. The vector direction of these Lorentz forces is parallel to the x-y plane of the cavity and do not oppose the positive z-directed Lorentz forces exerted by the magnetic field on the AC currents travelling in the top plate. Surface integration of all Lorentz forces (generated by the electric field and magnetic field of the EM wave) over the entire inner surface of the resonating cavity yields a time-averaged positive z-directed Lorentz force on the cavity. The net force vector is directed along the axial center of the cavity and directed from the bottom plate towards the top plate.

A resonating cavity can be modeled as an LC circuit. In the capacitive regions of the cavity, the electric field forces predominate on the cavity walls. In the inductive regions of the cavity, the magnetic field forces predominate on the cavity.

In the POC cavity, the magnetic field of the EM wave interacts with the cavity walls in the inductive region of the cavity. Here the top and bottom plates of the POC cavity are asymmetric. The magnetic field of the EM wave interacts with AC surface currents in this asymmetric region and creates forces on the surfaces. These forces are asymmetric, creating force vectors parallel to the x-y plane on the side walls of the slots of the bottom plate that do not exist on the top plate. On the top plate above the slots of the bottom plate, all magnetic field energy creates a Lorentz force with a z-directed component. On the bottom plate, substantial surface currents exist on the vertical walls of the slots. Magnetic field interactions on these AC currents create Lorentz forces with no z-directed component. The asymmetry in z-directed magnetic field Lorentz forces exerted on the top and bottom plates creates a time-averaged net force on the cavity.

In the capacitive region the POC cavity is nearly symmetric, causing the electric-field Lorentz forces on the cavity walls to create nearly symmetric (with opposite vector sign) Lorentz forces. The structure of the capacitive region is similar to a parallel-plate capacitor. Most of the electric field energy of the EM wave interacts with the symmetric parallel plates of the cavity walls. The z-directed electric-field-induced Lorentz forces are symmetric in the central region of the cavity. Near the equatorial walls of the cavity, the electric field of the EM wave is weak (compared to areas near the axial center of the cavity). The electric field of the EM wave acts asymmetrically on the cavity surfaces near the cavity equatorial walls (the inductive region of the cavity). The asymmetric geometry of the cavity at and above the slots of the bottom plate induce z-directed asymmetric interactions of the electric field with surface charges on the cavity walls. This asymmetric interaction creates a time-averaged, net force on the POC cavity that points in the negative z-direction. The time-averaged, net electric-field-induced force is much smaller in magnitude than the positive z-directed force created by the magnetic field interactions with this same asymmetric region of the cavity.

Most of the magnetic field energy interacts asymmetrically with the cavity. Most of the electric field energy acts symmetrically with the cavity. A time-averaged net Lorentz force (combined electric and magnetic field Lorentz forces) is generated by the interaction of EM wave energy with the cavity.

The use of numerical methods is required to calculate the time-averaged, net Lorentz force on the cavity. The axial asymmetry of the bottom plate requires the use of a three-dimensional FEM field solver in order to model the Lorentz forces on the cavity walls.

Numerical analysis of a variety of axially-symmetric SRF cavity geometries was performed using the 2D numerical method software SUPERFISH. Using SUPERFISH numerical results, it is shown that axially-symmetric cavities that are not symmetric with respect to the equatorial plane of the cavity (equatorially asymmetric) can sustain a time-averaged net magnetic-field-induced, z-directed Lorentz force. These forces are balanced by an equal and oppositely-directed, time-averaged, net electric-field-induced, z-directed Lorentz force. In an axially-symmetric, equatorially-asymmetric cavity, the E-field and H-field induced Lorentz force imbalances sum to zero. The net Lorentz force induced on an axially-symmetric resonant cavity is always zero.

Los Alamos National Laboratory website, http://laacg1.lanl.gov/laacg/services/download_sf.phtml#ps0

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Like an axially-symmetric, equatorially-asymmetric cavity, operation of the POC cavity creates a net time-averaged, magnetic-field-induced, z-directed Lorentz force. However, this magnetic-field-induced net force is not balanced by the electric-field Lorentz forces since the electric field predominantly interacts with the POC cavity in areas of high symmetry. A time-averaged, net z-directed Lorentz force on the cavity results because the net electric-field-induced Lorentz force is substantially lower in magnitude than the net magnetic-field-induced Lorentz force. On the POC cavity the sum of the E-field and H-field induced Lorentz force imbalance is not zero. Further details on the calculation of the time-averaged net Lorentz force are included in the numerical results section of this paper.

IV. Numerical Analysis

A. Numerical Results

Two Finite Element Method (FEM) numerical-method 3D electromagnetic-field-modeler programs, HFSS and Analyst were used to analyze the design of the POC cavity. The FEM is the preferred technique for eigenmode solutions of 3D EM field problems. Figure 5 depicts the pie-wedge-shaped model used for numerical method analyses. In Figure 5 the FEM model shape is oriented near a section of the vacuum space from which the wedge is taken. The model includes 1/2 of a slot and 1/2 of a bridge area from the vacuum space within the POC cavity. The shape represents a 2.5-degree-wide slice from the complete vacuum space within the cavity. The shape plus its mirror image comprise 1 complete bridge and two 1/2 slot sections on either side of the bridge. 72 pairs of these wedges comprise the complete vacuum space within the cavity making a total of 144 wedges comprising the vacuum space of the POC cavity. All 144 wedge shapes required to model the complete vacuum space within the POC cavity experience identical z-directed forces. By modeling the z-directed forces exerted on one 2.5 degree wedge shape, the z-directed force on the entire cavity can be determined.

The shape of the slot-cut in the FEM model shape is a rectangular prism. In the actual POC cavity, the slot cut shape is depicted in Figure 2 and is based on manufacturing considerations. The rectangular prism shape of the slot cut in the FEM model shape is used to simplify numerical analyses. This simplification will cause a small variation in numerical results compared to actual cavity performance. This variation is negligible with respect to the net z-directed force generated on the cavity.

In order to maximize the number of elements in the numerical solution, the signal ports on the top plate of Figure 2 were not modeled.

Figure 6 depicts the pie-wedge shape of Figure 5 with a mesh that was created in HFSS. The mesh in this image has 694,204 tetrahedral elements. The dark surfaces on the underside of the shape are due to a very dense mesh pattern on these surfaces. The mesh was seeded on the bottom and top conducting surfaces of the wedge to maximize the number of tetrahedrons on these surfaces. An identical pie-wedge shape was used in the FEM software Analyst. Numerous runs of the model were performed with meshes that produced in excess of 400,000

Figure 5. FEM model shape oriented within the POC cavity vacuum space

Figure 6. Tetrahedral mesh of the FEM model in HFSS
tetrahedral elements in the model (resulting in greater than 50 million tetrahedra for the cavity).

Results of the numerical method show that the cavity has a geometry factor, G of 100.\(^{11}\) The numerical results show that positive z-directed, magnetic-field Lorentz forces on the cavity top plate are approximately 7.5% greater than the magnitude of the negative-z-directed, magnetic-field Lorentz forces on the bottom plate. The differential in magnetic-field Lorentz-force pressure between the top plate and the bottom plate over the area of the slots is approximately 9%. The slots on the bottom plate are placed in an area of the cavity that experiences high magnetic fields and weak electric fields. There is a net negative-z-directed-force differential between the top plate and the bottom plate due to electric-field Lorentz forces. This force is much smaller in magnitude than the H-field net force since the cavity asymmetries are located in areas of low E-field and high H-field.

Calculation of the z-directed force: Due to cavity symmetry, all Lorentz forces parallel to the x-y plane cancel out. There are no net x-y directed forces developed on the cavity. The z-directed forces are calculated by taking the integral sum of all z-directed forces exerted on the inner cavity surfaces. At steady state, the time-average of all shear forces exerted on a surface element are zero. The time-averaged force exerted on a surface element is parallel to the surface normal of the surface element. The z-directed component of a force exerted on a surface element is calculated by multiplying the total force on the element by the dot product of the surface-normal vector and a unit vector that is parallel to the z-axis. The total z-directed force is the integral sum of all z-directed forces exerted on individual surface elements.

Figure 7 depicts several perspective views and a detail view of the FEM model shape of Figure 5. Surface 7 of Figure 7 is a symmetry plane (perfect H) of the FEM model shape. The second symmetry plane (perfect H) is located on the opposite side of the wedge. Detail A is an exploded view of the mill cut on the side wall of the slot. Only surfaces 1, 2, 3, 4, 5, and 6 of Figure 7 have surface-normal vectors with components parallel to the z-axis (and experience a Lorentz force). No other surfaces in the model will experience a time-averaged z-directed force.\(^*\)

\[ \text{The total z-directed force is calculated in HFSS using the Fields Calculator feature of the software. The magnetic field of the TM}_{010}\text{ wave exerts a positive z-directed force on surfaces 4, 5 and 6. The magnetic field exerts a negative z-directed force on surfaces 1, 2 and 3. The electric field exerts a z-directed force directed opposite to the magnetic field; a negative z-directed force on surfaces 4, 5 and 6 and positive z-directed force on surfaces 1, 2 and 3.} \]

The total, time-averaged z-directed force is:

\[
\frac{1}{2} w \int_{S_{z=\pm z}} \left( \mu_0 |H_{\text{max}}|^2 - \varepsilon_0 |E_{\text{max}}|^2 \right) \hat{z} \cdot \dd s.
\]

(5)

where:

\[ w = 144(2.2E7)^2 \]

and is a scaling factor

HFSS normalizes all field results to an E-field maximum of 1. The scaling factor \( w \) scales the E-field and H-field up to a 22 MV/m maximum E-field, and 100 kA/m maximum H-field and incorporates 144 to account for all of the wedge shapes required to comprise the POC cavity. For the superconducting Niobium POC cavity, the approximate maximum H-field sustainable prior to cavity quench is 100 kA/m.

At peak power, the POC cavity is predicted to generate a time-averaged, z-directed force of 0.58 newtons with peak surface current of 100 kA/M located along the intersection of the bridges and slot cuts of the bottom plate of Figure 2. Peak power also corresponds to a maximum E-field of 22 MV/m.

Additional analyses of the net z-directed Lorentz force generated by the POC cavity were performed using numerical analyses from Analyst with postprocessing in Microsoft Excel. These analyses produced numerical results for total cavity thrust comparable to the HFSS results.

The projected thrust level of 580 mN for the POC cavity is comparable to the thrust produced by existing ion propulsion systems used in satellite and space-probe applications. Ion thrust levels for NASA’s Deep Space 1

\[\text{** The flat face at the bottom of the slot experiences negligible E-field and H-field, so this surface is not included in Lorentz force calculations.} \]

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mission produced 90 mN of thrust.\textsuperscript{††} The Boeing Company uses xenon ion engines (XIPS) that produce 18 mN and 165 mN of thrust for their 601HP and 702 satellite architectures.\textsuperscript{‡‡}

Figure 8 depicts the H-field intensity on the FEM model shape at H-field maximum value. In Figure 8, dark blue represents field minimum value and green represents the highest intensity field on the wedge shape. At the bottom surface of the slot, H-field values are approximately zero.

\textbf{Figure 8. H-Field on the FEM model}

Figure 9 is a magnified view of the H-field on the slot walls at maximum H-field. The AC current that exists on the vertical slot wall interacts with the magnetic field of the wave and generates a time-averaged Lorentz force that is orthogonal to the z-axis. This force is parallel to the x-y plane of the cavity and does not exist on the top plate of the POC cavity. The vertical slot walls of the bottom plate divert some magnetic field energy onto areas of the cavity which create no z-directed Lorentz forces. This creates a differential in the total z-directed, H-field-induced Lorentz force on the cavity.

Figure 10 depicts a slice-plane view of the magnetic field around \( \frac{1}{2} \) of a slot and \( \frac{1}{2} \) of a bridge from a numerical method analysis performed with Analyst. The slice plane is located at a distance of 9 cm from the primary axis of the cavity. In this figure, the magnetic field is color-coded with blue as the lowest (zero field) and green as the highest magnetic field. Post processing shows that negative-z-directed, magnetic-field Lorentz forces on the bridges between the slots of the bottom plate are approximately 9\% less in magnitude than the positive-z-directed, magnetic-field Lorentz forces exerted on the top plate (in the area above the slots and bridges of the bottom plate).

\textbf{Figure 10. H-Field at a cross-sectional plane of the FEM shape}

Figure 11 depicts the E-field magnitude in the model at E-field maximum of the wave cycle. Red depicts the highest magnitude E-Field and blue depicts the minimum E-field. The magnitude of the E-field energy (at field maximum) in the cavity is equal to the magnitude of the H-field energy (at field maximum) in the cavity. Most of the H-field energy acts on the cavity in the asymmetric region of the cavity as depicted in Figures 8 and 9. Most of


\textsuperscript{‡‡} NASA Glenn Research Web Site, http://www.nasa.gov/centers/glenn/about/fs08grc.html
the electric field energy acts on the cavity in areas of high symmetry as depicted in Figure 11. The E-field interacts with the asymmetric areas of the cavity (the slots of the bottom plate) in areas where the E-field approaches zero magnitude. The H-field interacts with the POC cavity in the asymmetric areas where the H-field approaches maximum magnitude. The vector sum of the net H-field-induced force plus the net E-field-induce force yields a positive-z-directed net Lorentz force exerted by the EM wave on the cavity.

B. Summary of Numerical Results

Numerical method predictions for cavity Q are 1E9 at He temp of 2.3 K with peak net force levels of approximately 580 mN at peak surface currents of 100 kA/m and peak E-field of 22MV/m. Peak surface current is located on the edges of the bridges of the bottom plate. The net force vector is coincident to the z axis and points in the positive z direction. The geometry factor for the cavity is 100 Ohms. At peak power, field energy is 0.269 joules. Predicted cavity frequency is 1.054 GHz.

V. Experiment Results

A. Results

On January 13, 2011, the charts of Figures 12 and 13 were generated by sending 10.5 watt power pulses of 1047.335 MHz RF phase-locked power forward to the POC cavity. The POC cavity was operated in the TM_{010} mode. Figures 12 and 13 show dips (9 in Figure 12 and 6 in Figure 13) in the compressive force on the load cells. The dips in the load-cell outputs coincided with the 10.5 watt power pulses sent into the cavity from the signal generation circuit. The power pulses were each 4-5 seconds in duration. The voltage output of the load-cell circuit dropped when power was sent into the cavity. The voltage output of the load-cell circuit increased and moved back to the signal trend line after power from the signal generation circuit was turned off.

Figures 12 and 13 also show positive peaks in the voltage signal coming from the load cells. These positive peaks resulted from placing a calibrated 2-gram weight onto the support arm that supports the POC cavity and vacuum tubing. The duration of the test weight additions to the support arm was also 4-5 seconds.

The dips (9 of Figure 12 and 6 of Figure 13) in voltage correspond to a reduction in compressive force on the load cells of approximately 7-10 mN.

Based on further testing of the experimental apparatus, it was demonstrated that the liquid helium in the helium vessel was below atmospheric boiling temperature during the test runs captured in Figures 12 and 13. Both charts were generated less than 80 minutes after breaking the vacuum seal over the helium vessel of Figure 4 (where liquid helium was at equilibrium at 50 Torr). Formation of helium gas bubbles was not responsible for the response of the load cells during periods of EM power being sent into the POC cavity.

Additional test runs were performed on 2 subsequent test days with additional results demonstrating a reduction in compressive force on the load cells of the experimental apparatus during periods when phase locked EM power was sent into the POC cavity.

Figure 14 is a chart of experimental results from day 2 of testing. This chart was generated prior to pump-down of the liquid helium to 50 Torr. Helium temperature in the helium vessel was 4.2 K during the test run of Figure 14. This test was performed for comparative data between tests run with and without helium that was cooled below its atmospheric boiling temperature. In Figure 14, there are 3 smaller dips in voltage output which coincide with 10.5
watt power pulses being sent into the cavity. The 2 larger dips in voltage output from the load cell circuit correspond to removal and replacement of a 2 gram calibrated test weight from the support arm of the experimental apparatus. The duration of each of the power pulses and test weight pulses was approximately 4-5 seconds.

The 3 power-pulse dips of Figure 14 correspond to a reduction of compressive force on the load cells of approximately 7 mN. These dips are smaller in magnitude than the dips associated with the tests of Figure 12 and 13.

The cavity Q only marginally varied between values at 4.2 K and 2.3 K. Most power sent into the POC cavity was lost in non-BCS ohmic heating. It is believed that the signal port design and location are responsible for the non-BCS losses. The comparable voltage signal reductions in Figures 12, 13, and 14 correspond with the marginal Q improvement between the low temp (2.3-3 K) tests and the 4.2 K tests.

Numerical predictions for POC cavity Q values are approximately 1E9 with cavity temperature at 2.3 K. Experimental results measured Q of 1.08 E7. Niowave Inc. manufactured and chemically polished other SRF cavities from the same cylinder of solid 250 RRR niobium used to produce the POC cavity and the Q values of these other cavities are in the range of 1 E9.

B. Experimental Results Summary

At measured Q of 1.08 E07, power pulse-levels of 10.5 watts produce a stored energy of 7.73 E-03 joules. Field energy is calculated using forward power, reflected power, transmitted power, and decay time readings from the test runs. Numerical method predictions are for a 16.7 mN net Lorentz force to be generated on the POC cavity at this energy level. Experimental results demonstrate a 7-10 mN reduction in compressive force on the load cells when the field energy in the cavity is 7.73 E-03 joules. Tests on the cavity with helium at 4.2 K produced reductions in the compressive force on the load cells of approximately 7 mN.

VI. CONCLUSIONS

When operated with a TM_{010} EM wave, the POC cavity experiences a net-time-averaged, z-directed Lorentz force. FEM-numerical-method-analysis predictions for net Lorentz force on the POC cavity are within approximately 50% of experimental results. Variations between experimental results and numerical-method predictions are potentially due to:

- limitations of the experimental measurements of cavity Q
- limited data available calculations of stored energy levels achieved during test pulses
- load-cell sensitivity at the measured force levels

EM field energy in the POC cavity during experimental runs was less than 3% of design value which limits correlations between numerical-method predictions and experimental results.

Although the alternative scenarios explaining the load-cell circuit response are remote, further experimentation will be used to definitively rule out liquid-helium effects as the cause of the experimental results. Cannae plans to test a new, larger cavity with improved geometry and improved signal-port design. The numerical analysis of this new cavity shows that the unbalanced force levels generated by the new design far exceeds the lift achievable by non-equilibrium helium flow and/or localized helium boiling on the cavity walls.

This new propulsion technology represents a significant breakthrough in satellite and deep space propulsion. The elimination of on-board propellant opens up new and improved mission opportunities while significantly reducing mission-life costs for space vehicles.

Appendix: Review of Potential False Positives and Cross-Check of Numerical Results

The POC experiment is designed to measure a time-averaged Lorentz force generated on the POC cavity. The net Lorentz force on the cavity is measured with two load cells that support the entire weight of the cavity and attached vacuum piping. A variety of factors which are unrelated to a net Lorentz force have the potential to affect
load-cell output. These potential sources for false positive signal generation were minimized by the experimental
design and experimental procedure. An examination of potential sources for false positives follows.

A. Helium buoyancy

Ohmic heating occurs on the walls of the POC cavity during experimental runs. The heat generated in test pulses is
removed from the cavity walls by heat transfer to the liquid helium bath. Helium gas bubble formation beneath the
cavity could cause false positive signals in the load-cell circuit. In order to eliminate helium gas bubbles as a
possible source of false signals, helium temperature was reduced to 2.3 K by reducing the pressure over the liquid
helium to 50 Torr. At equilibrium, 50 Torr vapor pressure over the liquid helium corresponds to a liquid temperature
of 2.3 K. Further reductions in the liquid helium temperature were not done in order to prevent the helium from
becoming superfluid.

After successful test runs on Day 3 of testing, power was sent continuously into the POC cavity in order to
induce boiling of the liquid helium in the helium vessel. At 55 minutes after breaking the vacuum seal over the
liquid helium (helium at 50 Torr vacuum pressure while vacuum-sealed), continuous power forward of 10.5 watts of
phase-locked power was sent into the cavity. Cloud was observed over the helium vessel opening after
approximately 20.5 minutes of continuous 10.5 watt input power into the cavity. The cloud at the helium vessel
throat opening indicated that liquid helium was boiling in the helium vessel. The energy required to bring the liquid
helium to boiling temperature (after 55 minutes and experimental runs with 4-5 second 10.5 watt test pulses) was
12,915 joules.

All tests on the cavity were performed within 80 minutes of breaking the vacuum seal over liquid helium at 2.3
K. The test dewar has an estimated heat transfer rate into the liquid helium bath of approximately 1 watt. Based on
the amount of energy required to boil helium after the tests of Day 3, all experiments performed (after liquid helium
was cooled to 2.3 K) did not produce enough energy to boil the liquid helium. Based on dewar heat transfer rates
and energy input from test pulses, all low-temperature-helium tests were performed with liquid helium between 2.3
K and 3 K. Boiling helium bubbles under the POC cavity did not produce the reduction in compressive forces on the
load cells which occurred when power pulses were sent into the POC cavity.

B. Circuit Cross-Chatter

After 2 successful experimental runs, 10.5 watt pulses of EM energy were sent forward to the POC cavity.
These power pulses were not phase locked. No response was recorded by the load-cell circuit when non-phase-
locked power was sent to the cavity. This test was performed to verify that circuit cross chatter between the signal
generation equipment and load-cell circuit did not create false positive signals in the load-cell circuit output.

C. Load-Cell Thermal Isolation

The load cells are located in the proximity of the throat of the helium vessel (Figure 4). In order to isolate the
load cells from cold gases emitted from the helium vessel throat, neoprene sheeting was suspended between the
load-cell platforms and the helium vessel throat. The neoprene sheeting did not mechanically connect the support
arm to the load cells or load-cell platform. The thermal isolation of the load cells from the throat gases prevented
temperature fluctuations of the load cells from affecting the load-cell output.

In addition to the neoprene insulation, Teflon barriers were placed above and below the load cells to thermally
insulate the load cells from metal-to-metal contact with the support arm and the load-cell platforms.

D. Load-Cell Performance Limits

The POC cavity stored energy level achieved in the experiment was significantly lower than numerical method
predictions for maximum field energy due to low cavity Q. Correspondingly, the maximum net Lorentz force levels
achieved in the experiment were significantly lower than numerical predictions. The load cells used in the
experiment operated near the limit of their sensitivity. Prior to the experimental tests, numerous calibration runs on
the load-cell circuit were performed on a mock-up of the support arm with attached piping and cavity. During the
calibration runs, the load-cell circuit routinely and clearly responded to 2 mN load removals from the mock support
arm. The load-cell compressive force reduction measurements of the POC experiment were near the limit of load-
cell circuit sensitivity, but still 3.5 times greater than the demonstrated minimum force measurable by the circuit.

E. Alternative Explanations for Load-Cell Response

The POC test apparatus is designed to measure the differential in radiation pressure exerted on the POC cavity.
Alternative possibilities (to a radiation pressure imbalance) exist for the creation of dips in the voltage output that
occurred when power pulses were sent into the POC cavity.
Alternative 1) An asymmetric heat flow on the cavity walls (due to the asymmetric cavity shape) caused asymmetric helium flow which created a temporary signal spike in the load-cell-circuit output. This possibility is remote since the power-pulse lengths were 4-5 seconds. The voltage signal reduction during the power pulses remained reduced until the power pulse was turned off. The voltage reduction of the power pulse responded like a square wave. Given the power-pulse length and the shape of the voltage drop, non-equilibrium helium flow on the cavity is unlikely to have created the load-cell-circuit response to power pulses.

Alternative 2) Localized-helium boiling created lift underneath the cavity which created the voltage reduction pulse. This possibility corresponds with the heat energy that was generated during the power pulses. Tests on the cavity were performed with liquid helium between 2.3 and 3 K. Localized-helium boiling on the underside of the cavity could have been responsible for creation of lift beneath the cavity if the heat transfer rate from the helium at the cavity walls to the helium bath was insufficient to disperse the localized heating. This scenario is also unlikely. The chart of Figure 14 was generated with helium at atmospheric boiling temperature. The voltage response of the load-cell circuit produced pulses corresponding to a 7 mN reduction of pressure on the load cells. If boiling helium caused the lift measured in Figures 12 and 13, the effect would be more pronounced in Figure 14. Figures 12, 13, and 14 all produce comparable load-cell voltage output reductions.

In addition, the primary energy loss mechanism in the POC cavity is likely associated with the signal port placement and design. Helium bubble formation at the signal ports would not produce lift on the cavity. The ohmic losses associated with the BCS resistance of the superconducting niobium are projected to be .063 watts at the field levels achieved in the experiment. If all of the BCS-induced ohmic heating occurred on the bottom plate of the cavity, it could not generate enough helium gas to create the 7-10 mN load-cell response. If all of the BCS-related ohmic heating (from a 5-second pulse) generated trapped helium gas beneath the cavity, the total buoyancy effect on the load cells would be less than 1 mN.

F. Cross-Check of Numerical Results

The numerical results were cross-checked by four methods.

Cross-check #1: Two separate FEM software programs were used to model the unbalanced force generated by the cavity. Numerous separate analyses (based on separate mesh, convergence and solution-order criteria) were performed on the cavity and all analyses showed an unbalanced force between the top and bottom plate that exceeded 5% of the total z-directed H-field-induced force on the top plate.

Cross-check #2: In HFSS simulations, the ratio of the electric field energy and the magnetic field energy was compared to get an order of magnitude estimate of the error in the numerical results. The ratio of E and H energies (at maximum respective field values) are calculated in the HFSS Fields Calculator using the formula:

\[ r_E = \frac{\int \mu_0 H_z^2}{\int \varepsilon_0 E_z^2} \]

The \( r_E \) ratio should be 1 for all resonating cavities. All HFSS simulations on the POC cavity had an H/E energy ratio within 0.1% of 1.

Cross-check #3: In order to check the accuracy of the surface-integral calculation used in HFSS, standard-design elliptical cavities were modeled. Z-directed forces induced by the H-field were compared between the upper and lower surfaces of the cavities. The ratio of the z-directed forces is calculated as

\[ r_H = \frac{\int_{S_{upper}} \mu_0 H_z |H_z| n \cdot z}{\int_{S_{lower}} \mu_0 H_z |H_z| n \cdot z} \]

where

- \( r_H \) is the dimensionless ratio of z-directed, H-field forces on the upper half of the cavity divided by the z-directed, H-field forces on the lower half of the cavity
- \( S_{upper} \) is the inner surface of the upper half of cavity
- \( S_{lower} \) is the inner surface of the lower half of the cavity
- \( n \) is the surface normal vector
- \( z \) is the unit vector parallel to the z axis
Due to cavity symmetries of an elliptical cavity, $r_{zH}$ should be 1. In HFSS simulations performed on standard elliptical cavity geometries, all calculated ratios are within 0.1% of 1.

Cross-check #4: A hypothetical cavity created by uniting 2 identical bottom plates (Figure 2) of the POC cavity was simulated in HFSS. A pie-shaped wedge that is a 2.5 degree slice of the vacuum space within the hypothetical cavity was used in the numerical analysis. The model shape used in the FEM analysis is depicted in Figure 15. This wedge represents a 1/144th section of the hypothetical cavity. Due to symmetry, $r_{zH}$ of Equation 8 should be 1 for this cavity. A meshing of 463,369 tetrahedra on the shape of Figure 15 produced an $r_{zH}=0.99968$.

For the numerical analysis of the shape in Figure 15, the upper surface of the mill cut on the lower slot is included in the integral of the numerator of $r_{zH}$. The lower surface of the mill cut on the upper slot is included in the integral of the denominator of $r_{zH}$.

Table 1 is a comparison of numerical data between the POC cavity and the hypothetical cavity of Figure 15. Row 2 of the data table shows the error percentage ratio between the electric field energy and the magnetic field energy. In all resonating cavities, this error percentage should be zero. The numerical analysis of both cavities have an error in the H and E-field energy ratio of less than .1 %.

Row 3 of the data table is the percentage difference between the positive and negative z-directed H-field-induced Lorentz force on each cavity. Due to symmetry, this value should be zero for the cavity of Figure 15. The numerical analysis of the cavity produced a value within 0.033 % of a zero value. The value of 7.5 % differential in H-field-induced Lorentz force on the POC cavity is significantly outside of the error values of all other numerical results.

When the E-field induced imbalance is added to the H-field-induced imbalance, the total unbalanced force on the POC cavity is 6.35% of the total H-field-induced, z-directed force on the top plate. The numerical result for the net Lorentz force generated on the POC cavity is significantly outside of the margin of error for all other cavities and calculations.

<table>
<thead>
<tr>
<th>Table 1. Numerical results comparison of POC cavity to Figure 15 cavity</th>
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<tr>
<td>Tetrahedra (2.5 degree section)</td>
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<tr>
<td>-----------------------------------------------</td>
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<tr>
<td>(1-$r_x$) x 100</td>
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<td>(1-$r_{zH}$) x 100</td>
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Cannae LLC has patent-pending status on the technology described in this paper.

References


