

Critical geometry of a thermal big bang

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We explore the space of scalar-tensor theories containing two nonconformal metrics, and find a discontinuity pointing to a “critical” cosmological solution. Due to the different maximal speeds of propagation for matter and gravity, the cosmological fluctuations start off inside the horizon even without inflation, and will more naturally have a thermal origin (since there is never vacuum domination). The critical model makes an unambiguous, nontuned prediction for the spectral index of the scalar fluctuations: $n_s = 0.96478(64)$. Considering also that no gravitational waves are produced, we have unveiled the most predictive model on offer. The model has a simple geometrical interpretation as a probe 3-brane embedded in an $EAdS_2 \times E_3$ geometry.

DOI: [10.1103/PhysRevD.94.101301](https://doi.org/10.1103/PhysRevD.94.101301)**I. INTRODUCTION**

In spite of its mathematical simplicity and observational triumphs, the big bang model of the Universe remains an unfinished work of art. Many of its late-time successes can be traced to the initial conditions *postulated* for its early stages, and these are put in by hand, without justification, other than to retrofit the data. The main culprit for this shortcoming is the so-called horizon problem: the cosmological structures we observe today span scales that lay outside the ever-shrinking “horizons” of physical contact that plagued the early Universe. This precludes a causal explanation for their initial conditions.

Several extensions of the big bang model have been proposed with the aim of opening up its horizons. An early bout of accelerated expansion [1–3], a contracting phase followed by a bounce [4], a loitering early stage [5], and a varying speed of light (VSL) [6,7] have all been considered. None of these proposals evades the criticism that retrofitting the data is still used to select *in detail* the primordial fluctuations that the model *should* produce. Once primordial causal contact is established, work can start on concrete physical mechanisms for spoiling perfect homogeneity (e.g. vacuum quantum fluctuations or thermal fluctuations). Typically it is found that one can produce a wide range of initial conditions including, but not circumscribed to, those explaining the observations.

Specifically, the primordial density fluctuations can be characterized by an amplitude A_s , measuring their intensity at a given scale, and a spectral index n_s , measuring how the amplitude varies with scale. Observations [8,9] show that $A_s = 2.142(49) \times 10^{-9}$ and $n_s = 0.9667(40)$, signaling a very specific slightly red spectrum, i.e. one with enhanced amplitude for longer wavelengths. While

the observed A_s probably indicates nothing more than a hierarchy between two energy scales, one might have expected a theoretical forecast for n_s . Yet, all theories effectively adjust their free parameters (e.g. the reheating temperature after inflation, or the number of e -foldings) to fit the observed n_s , from within a range of possibilities. This is not to say that they entirely lack predictivity; indeed they do predict a plethora of conditions involving n_s and other observables (e.g. [10]).

In this article, we revisit a class of VSL models [11] in which there are two nonconformal metrics, one for matter and another for gravity, so that light and other massless matter particles travel faster than gravity. Conditions for the observational success of such models have been identified [10,12–16], considering both a vacuum and a thermal initial state. In common with other models, they do not bypass the criticism voiced above. However, in this article we uncover a remarkable result pertaining to thermal scenarios.

It is known that thermal VSL models require a fast phase transition in c so as to produce near-scale-invariant fluctuations; however, the scale-invariant limit ($n_s = 1$) is unreachable. Closer inspection of the space of all possible theories reveals that this is due to a discontinuity, pointing to a special, critical solution that should be regarded as the preferential model for a phase transition in c . Instead of $n_s = 1$, the thermal fluctuations in this model display a running $n_s < 1$. But what is truly notable is that the model has a single free parameter, so that the amplitude A_s fully fixes the value of n_s at the observationally relevant scales. The predicted value is within current constraints, but improved observations would unambiguously prove or rule out the theory. The model also has a simple geometrical interpretation as a probe 3-brane embedded in an $EAdS_2 \times E_3$ geometry.

II. THE CRITICAL MODEL

We start by reviewing the general framework of scalar-tensor bimetric theories. In these models there are two metrics (or frames): $g_{\mu\nu}$ associated with the gravitational action (the Einstein frame), and $\hat{g}_{\mu\nu}$, to which matter is minimally coupled (the matter frame). The action takes the general form,

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} R[g_{\mu\nu}] + \int d^4x \sqrt{-\hat{g}} \mathcal{L}_M(\Psi, \hat{g}_{\mu\nu}) + S_\phi,$$

where M_P is the reduced Planck mass, and S_ϕ encodes the dynamics of the field ϕ relating the two metrics. If the metrics are conformally related we have a “varying- G ” theory, such as Brans-Dicke theory. In “varying- c ” theories, rather, the metrics are nonconformally related,

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B(\partial_\mu \phi)(\partial_\nu \phi), \quad (1)$$

so that the light cones spanned by massless matter particles and by gravitons do not coincide [13,14]. In general B (also known as the *warp factor*, for reasons to be made obvious soon) is a function of ϕ . If the speed of light is to be larger than that of gravity, then $B > 0$ (with signature $+- --$).

It may seem that the number of theories of this type is endless, but this is not the case. The simplest nontrivial S_ϕ must consist of two generally nonconstant cosmological terms, one in the matter frame and the other in the Einstein frame,

$$S_\phi = \int d^4x \sqrt{-\hat{g}} (-2\Lambda_m(\phi)) + \int d^4x \sqrt{-g} (-2\Lambda_g(\phi)). \quad (2)$$

Furthermore, only one of the functions $\Lambda_m(\phi)$, $\Lambda_g(\phi)$ and $B(\phi)$ is independent [17]. This has been known for a while; here we sketch the proof in two steps.

First, let ϕ be canonically normalized in the generalized sense that it should exhibit a Klein-Gordon equation of motion in the matter frame when no matter is present. Then, its action should be a cosmological term in the matter frame, due to a simple calculation in variational calculus in the presence of two metrics (see [13,14]). Furthermore, if the field dynamics is to be driven by $2\Lambda_m$, regarded as a potential, we should have $2\Lambda_m(\phi) = -1/B(\phi)$. This fully fixes the first term of (2) in terms of B , and it is known [14] that it amounts to postulating a Dirac-Born-Infeld (DBI) action in the Einstein frame

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{B} \sqrt{1 + 2BX} - V \right), \quad (3)$$

with $X = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)$ and $V = 2\Lambda_g$. This can be derived from simple properties of determinants, and will be

important in seeking a geometrical interpretation for our critical model.

Secondly, of the two remaining free functions (B and V) only one is free in the UV limit, which is the limit of interest to us. In the physical situation we are considering, the action (3) should more strictly be called anti-DBI, since the sign of B is opposite to the usual one, so that the speed of light is larger, rather than smaller than that of gravity. Thus, the UV limit of the theory is achieved with $X \gg 1$ (instead of saturating at an upper bound, as is the case with the usual DBI theory), so that

$$\mathcal{L}_\phi \approx \sqrt{\frac{2X}{B}} - V + \mathcal{O}\left(\frac{1}{\sqrt{B^3 X}}\right). \quad (4)$$

This is nothing but the cusciton model [18,19], and indeed the speed of sound is infinite in this limit ($c_s \approx \sqrt{2BX} \rightarrow \infty$). The model has conformal (Weyl) symmetry [20,21], so that any scale-factor $a(t)$ is a solution. This implies that spatial flatness is compulsory and fully fixes V [18–20]. If ρ and p denote density and pressure, we have $\rho \approx V$ and $p + \rho \approx K$ [18,22], where $K = \dot{\phi}/\sqrt{B}$ is the kinetic energy. The (spatially flat) Friedmann and continuity equations are

$$3M_P^2 H^2 \approx V \quad \text{and} \quad \dot{V} + 3HK \approx 0, \quad (5)$$

where $H = \dot{a}/a$. These can be integrated as

$$V(\phi) = \frac{3}{4M_P^2} \left(\int \frac{d\phi}{\sqrt{B(\phi)}} \right)^2 + \mathcal{O}\left(\frac{\epsilon V}{c_s^2}\right), \quad (6)$$

fixing V as a function of B . Here $\epsilon = -\dot{H}/H^2 = \frac{3}{2}(1+w)$, where $w = p/\rho$, and c_s is the speed of sound in the matter frame. Although any w is possible, it can be shown that as $c_s^2 = \partial p / \partial \rho|_X \rightarrow \infty$, the next order corrections yield $w = p/\rho \rightarrow \infty$ for generic solutions [23,24].

So far we have merely reviewed old results. Now we come to the crucial element of this article. It has been shown [10,12,16] that thermal bimetric scenarios are close to scale invariance whenever $B(\phi) \propto \phi^n$, with n close to 2. Then, the potential V is still a power law, but its exponent is close to 0 [cf. Eq. (6)]. The variation in c_s is abrupt, but one still has constant $\epsilon_s = \dot{c}_s/(c_s H)$, with $\epsilon_s \rightarrow -\infty$ as $n \rightarrow 2$. The cosmological solutions are “scaling solutions,” i.e. they have constant ϵ and ϵ_s , leading to thermal fluctuations with constant n_S , which can be tuned to be as close to 1 as wanted. Indeed

$$n_S - 1 = \frac{\epsilon + 1}{\epsilon_s + \epsilon - 1}, \quad (7)$$

and although $n_S = 1$ is unreachable, any red spectrum as close to scale invariance as required can be obtained by suitably tuning B .

It should be immediately obvious from Eq. (6) the reason why the scale-invariant limit cannot be reached. Within the space of these theories, there is a discontinuity at $B \propto \phi^2$, because the potential fails to be a power law. All the theories around it imply power-law potentials, but this critical theory stands out as an exception,

$$B_{\text{crit.}}(\phi) = B_0 \left(\frac{\phi}{M_P} \right)^2, \quad (8)$$

$$\Rightarrow V_{\text{crit.}}(\phi) = \frac{3}{4B_0} \ln^2 \left(\frac{\phi}{M_P} \right). \quad (9)$$

It marks a special, crucial boundary in the space of theories. The critical model is unique in that it cannot have a constant ϵ_s , since V is no longer a power law. This induces natural deviations from scale invariance, making its phenomenology remarkable, as we show presently.

III. GEOMETRICAL INTERPRETATION

Before embarking upon the phenomenology of the critical model we reinforce its special status by uncovering an elegant geometrical interpretation. It is known that the DBI action can be derived from the induced metric on a probe 3-brane embedded in a higher dimensional geometry, with the B function interpreted as a geometrical “warp” factor. For example, in the celebrated DBI action associated with the motion of a probe 3-brane in $\text{AdS}_5 \times S_5$ geometry, one finds $B \propto -\phi^{-4}$, with interesting cosmological implications [25,26].

Likewise, $B(\phi) \propto \phi^2$ follows from embedding a 3-brane in the $E\text{AdS}_2 \times E_3$ geometry given by

$$d\tau_5^2 = \frac{r^2}{R^2} dt^2 + \frac{R^2}{r^2} dr^2 - dx^2 - dy^2 - dz^2, \quad (10)$$

where R is the radius of the Euclidean AdS_2 . Ignoring the gravitational backreaction, the induced action on a uniform probe 3-brane at $r(t)$ is given by

$$S_{3B} = T_3 \int d^4x \sqrt{\frac{r^2}{R^2} + \frac{R^2 \dot{r}^2}{r^2}}, \quad (11)$$

where T_3 is the brane tension (with mass units M^4). Field ϕ is a redefinition of r that renders (11) canonical in the IR limit, and a Taylor expansion shows that this is given by $r = 4R^3 T_3 / \phi^2$. Straightforward algebra shows that this brings (11) to the anti-DBI form (3) with B matching the critical model (8) and

$$B_0 = \left(\frac{M_P}{2RT_3} \right)^2. \quad (12)$$

Turning on gravity for the effective four-dimensional geometry, the potential is fixed by Eq. (6), as a result of the conformal invariance of the theory in the UV [20,21].

A crucial novelty here is that the extra dimension, r , is timelike rather than spacelike, something also discussed in string theory literature [27]. While this may raise alarm about ghost instabilities for the bulk, the ghost degrees of freedom may be made arbitrarily heavy and thus decouple from the four-dimensional low-energy effective field theory [24].

IV. DENSITY FLUCTUATIONS

We now come to the core of this article, the evaluation of the thermally induced fluctuations for the critical solution. This can be done following well-known methods developed for theories with a varying speed of sound c_s [14,16,22], since that is what our theory is in the Einstein frame. The second order action for the curvature fluctuation ζ is

$$S_2 = \frac{1}{2} M_P^2 \int d\eta d^3x z^2 [\zeta'^2 - c_s^2 (\nabla \zeta)^2], \quad (13)$$

where $z = \frac{a}{c_s} \sqrt{2\epsilon}$ and η is conformal time. Therefore we have a standard quantum field theory in variable $v = M_P z \zeta$, subject to dynamical equation

$$v'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) v = 0, \quad (14)$$

where k is the comoving wave number. The central quantity to be computed is c_s , and this is given by [14,18,22],

$$c_s = \sqrt{1 + 2BX} \approx \frac{2}{3} \epsilon B \rho \approx \frac{2}{3} \epsilon B_0 \rho e^4 \sqrt{\frac{B_0 \rho}{3}}, \quad (15)$$

where the first identity is generic for (anti-) DBI models, in the second step we used $\frac{2}{3} \epsilon = \frac{K}{V} \approx \frac{K}{\rho}$ and $K \approx \sqrt{2X/B}$, and in the third we used Eqs. (8) and (9). The fact that V (and so ρ) is not a power law in ϕ explains why the model has a c_s which is not a power law in a or ρ . Even if the background scales (constant ϵ), the speed of sound does not, with a varying ϵ_s given by

$$\epsilon_s = \frac{\dot{c}_s}{c_s H} = -2\epsilon \left(1 + 2\sqrt{\frac{B_0 \rho}{3}} \right). \quad (16)$$

Thus, n_s is expected to run, a property that can be guessed from (7). However that formula is incorrect for varying ϵ_s ; indeed many standard formulas in the literature [14,16,22] break down. A full derivation of n_s can be found in the Supplemental Material [23] (SM). Here we present an approximate calculation, good enough to extract all the salient features.

As usual, Eq. (14) has two regimes, an acoustic one and a gravitational instability one, depending on which of its two terms in v dominates. The two regimes are separated by the sound horizon scale, k_h , where these terms become of the same order,

$$c_s^2 k_h^2 \sim \frac{z''}{z} \approx (aH\epsilon_s)^2 - (aH\epsilon_s)' \approx (aH\epsilon_s)^2 \quad (17)$$

(for simplicity, we have assumed a constant ϵ , but in fact this is not necessary). The sound horizon scale therefore satisfies $c_s k_h \approx aH\epsilon_s$, and we note the extra factor of ϵ_s with regards to the usual formula. Matching the two types of solutions is sufficient to derive to a good approximation the power spectrum frozen in outside the horizon. For $k \gg k_h$ the solutions should be normalized as [14,16,22]

$$v = \frac{e^{-i \int c_s k d\eta}}{\sqrt{2c_s k}}, \quad (18)$$

whereas for $k \ll k_h$ the growing mode takes the form $v = F(k)z$. By means of simple algebra $F(k)$ can be found by matching the two expressions at $k \sim k_h$.

The square of $F(k)$ is nothing but the frozen-in power spectrum of ζ , up to a factor representing the expectation value of $2\hat{N} + 1$, where \hat{N} is the number operator (note that upon quantization v^2 is multiplied by $\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger$, where \hat{a} is an annihilation operator). For vacuum fluctuations, this factor is simply 1, whereas for a thermal state it is twice the thermal occupation number of mode k in the Rayleigh-Jeans limit [14,16]. This is $2T_c/k$, where $T_c = Tc_s/a$ is a “conformal temperature” which remains constant during the varying- c phase [14,16]. The frozen-in dimensionless power spectrum of the thermally induced fluctuations is therefore

$$\mathcal{P}_\zeta^{\text{th}}(k) \equiv \frac{k^3}{2\pi^2} \langle |\zeta|^2 \rangle^{\text{th}} \approx \frac{1}{24\pi^2} \frac{\epsilon_s^2}{\epsilon} \frac{\rho}{c_s M_p^4} \frac{T_c}{k}, \quad (19)$$

where the right-hand side is to be evaluated at horizon crossing (now $c_s k \approx aH\epsilon_s$). We stress the extra factors in ϵ_s found in Eqs. (17) and (19), in relation to standard formulas [22]. They are irrelevant if ϵ_s is a constant, but not in our case. Equation (19) is valid up to factors of order 1 (fully restored in the SM [23]).

Combining Eqs. (17) and (19) and using the chain rule we find for the spectral index

$$n_s - 1 = \frac{d \ln \mathcal{P}_\zeta^{\text{th}}}{d \ln k} = -\frac{1 + 2\epsilon}{4\epsilon} \left(\frac{B_0 \rho}{3} \right)^{-\frac{1}{2}} + \mathcal{O} \left(\frac{1}{B_0 \rho} \right),$$

where ρ is the density when $k = k_h$. Thus, n_s runs from very red, at the largest scales, to almost scale invariant, at the smallest. However, as announced, the observed amplitude A_s fixes where we are in this running flow. Note that the model has a single free parameter, the four-volume scale B_0 . Although Eq. (19) seems to depend both on $B_0 M_p^4$ and $B_0 \rho$, the former can be eliminated by using $c_s k = aH\epsilon_s$, the Friedman equation, and some basic thermodynamics, to recast it in the descriptive form,

$$\mathcal{P}_\zeta^{\text{th}}(k) \frac{g_0 T_0^3}{k^3} = C(B_0 \rho)^{1+\frac{2}{\epsilon}} \exp(4\sqrt{3}B_0 \rho), \quad (20)$$

where C is a numerical constant [23]. The left-hand side can be evaluated from observations. For a given mode (say, $k = 0.05 \text{ Mpc}^{-1}$) the first factor is the observed amplitude [$\mathcal{P}_\zeta(k) = 2.142(49) \times 10^{-9}$, [8]], and the second is the dimensionless entropy inside the scale k nowadays (with $g_0 = 3.91$ being the effective number of relativistic degrees of freedom). As stated above (and in [23]), generically $\epsilon \rightarrow \infty$, so we can solve (20) to get $B_0 \rho \approx 583.03(16)$, where the uncertainties are both observational and arise from the fact that the model is only reliable to $\mathcal{O}(\frac{1}{B_0 \rho})$. Using (20) we thus obtain

$$n_s = 0.96478(64), \quad (21)$$

well within the most stringent current observational constraints [viz. $n_s = 0.9667(40)$, cf. [8]].

This is a remarkable result. But the model makes further predictions. It produces no tensor modes (since the horizon problem is not solved for gravitons), and so singles out a point in the $\{n_s, r\}$ diagram, with $r = 0$. It also predicts [cf. Eqs. (17), (19) and (20)] the running of the spectral index to be

$$\frac{dn_s}{d \ln k} = -\frac{3}{2}(n_s - 1)^2 \approx -1.8 \times 10^{-3}, \quad (22)$$

within the allowed observational range of $(-6.5 \pm 7.6) \times 10^{-3}$ (see [8]). As for non-Gaussianity we find an amplitude for the bispectrum of order unity, $f_{NL} = \mathcal{O}(1)$, comparable to similar models [10] but with a very different and unique shape (to be reported elsewhere [24]). We have unveiled the most predictive model on offer.

What are the provisos of our claims? From the above we can work out that $B_0 M_p^4 \approx 6.6 \times 10^{13}$, so that the energy scale at the end of the transition is $3.5 \times 10^{-4} M_p$, with the current horizon scale leaving the sound horizon less than 3 orders of magnitude above this. So we never exceed the Planck scale (in common with other thermal varying- c scenarios [14,16]), allaying the first obvious criticism. Then, there are model uncertainties. The equation of state can have an effect on the final result (for example, $\epsilon = 2$ would push n_s down to $n_s = 0.95292$); however we have arguments for why $\epsilon \gg 1$ is generic in our model [23]. Furthermore, in evaluating T_c at horizon crossing we have assumed entropy conservation in the constant- c phase (the change in g drops out of the final result), but more importantly we have assumed no “reheating” at the end of the varying- c phase. This is because in our scenario any such process would be *ad hoc* and unnecessary, since the Universe is always hot. Nonetheless, we note that a reheat by a factor of, for example, 10^{10} would push n_s up to

$n_s = 0.96838$. An isothermal gluing of the two phases remains the most minimal assumption.

V. DISCUSSION

In summary, we built upon previous work on thermal fluctuations in bimetric scenarios which showed that a sufficiently fast phase transition in c_s leads to fluctuations as close to scale invariance as seen in the data [14,16]. In such scenarios, fitting the observed n_s requires fine-tuning the warp factor $B(\phi)$. Here we improved on this by discovering that the reason why exact scale invariance is never achieved is that the limit is discontinuous, pointing to a critical solution with quadratic warp factor, but a nonpower-law potential (fully determined by the Bianchi identities and UV conformal symmetry). The critical solution has a simple geometrical interpretation as the (anti-) DBI action of a probe 3-brane embedded in an $E\text{AdS}_2 \times E_3$ geometry. The nonpower-law nature of the potential induces a nonscaling speed of sound, which in turn produces a natural red tilt and running of the power spectrum.

But what makes the model remarkable is that the amplitude A_s for a given scale fixes its location on this overarching structure, leading to a single prediction for the observed n_s . The model does not require reheating, and this is the ultimate reason why it is more predictive than inflation, even if factors external to cosmology were to preselect one of inflation's many models. Inflationary models invariably predict a range of n_s , depending on the number of e -foldings, or the reheating temperature (even for a fixed choice of inflaton action). Adding to this the fact that our model makes precise predictions for the level of *primordial* gravitational waves ($r = 0$), the running of n_s , and non-Gaussianity, we can conclude without prejudice that we have in hand a very predictive model indeed. The fact that its main prediction (for n_s) lies spot in

the middle of the Planck results should not beguile us into a false sense of security. Improved observations will soon vindicate or disprove this model.

One may wonder about the status in our model of the other cosmological problems, such as the flatness, homogeneity and isotropy problems. First, the view may be held that such historically motivating problems are now considered to be of lesser importance than explaining the structure of our Universe, or may even be misguided [28]. Nonetheless we remark that it is possible to solve them using the VSL mechanism *before* the phase transition (e.g. [7,13]). In other scenarios it may also happen that their solution takes place in a different phase to structure formation. Furthermore, we find that at least the flatness problem can be solved, in a single package, during the phase transition. The conformal symmetry of the theory in the UV [20,21] not only fixes the potential but requires exact flatness [Eqs. (5) lead to a contradiction in the presence of spatial curvature]. A full investigation of these matters is deferred to [24].

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